VISVESVARAYA NATIONAL INSTITUTE OF TECHNOLOGY, NAGPUR
DEPARTMENT OF MATHEMATICS

Assignment-Vector calculus MAL-102

1. In what direction from (3,1,-2) is the directional derivative of $\phi = x^2y^2z^4$ maximum and what is its magnitude?

2. Evaluate $\int_c 2xds$ where c consists of arc c_1 of the parabola $y = x^2$ from (0,0) to (1,1) followed by the vertical segment c_2 from (1,1) to (1,2).

3. Evaluate $\int_c y^2dx + xdy$ where
(a) $c = c_1$ is the line segment from (-5,-3) to (0,2).
(b) $c = c_2$ is the arc of the parabola $x = 4 - y^2$ from (-5,-3) to (0,2).

4. Find the work done by the force field $\overline{F}(x, y) = x^2\ \mathbf{i} - xy\ \mathbf{j}$ in moving a particle along the quarter circle $\overline{r}(t) = \cos t\ \ \mathbf{i} + \sin t\ \ \mathbf{j}, 0 \leq t \leq \frac{\pi}{2}$.

5. (a) If $\overline{F}(x, y) = (3 + 2xy)\ \mathbf{i} + (x^2 - 3y^2)\ \mathbf{j}$ find the function f such that $\overline{F} = grad\ f$.
(b) If $\overline{F}(x, y, z) = y^2\ \mathbf{i} + (2xy + e^{3z})\ \mathbf{j} + (3ye^{3z})\ \mathbf{k}$ find the function f such that $\overline{F} = \bigtriangledown f$.

6. Evaluate (a) $\oint_c(3y - e^{\sin x})dx + (7x + \sqrt{y^4 + 1})dy$, where c is the circle $x^2 + y^2 = 9$.
(b) $\int_c(y^2\ dx + 3xy\ dy)$, where c is the boundary of the semi annular region D in the upper half plane between the circles $x^2 + y^2 = 1$ and $x^2 + y^2 = 4$.

7. Find the directional derivative of the function $\phi(x, y, z) = x^2 - y^2 + 2z^2$ at point $P(1, 2, 3)$ in the direction of the line PQ, where Q is the point (5,0,4). In what direction, the directional derivative will be maximum?

8. Show that $r^n\overline{r}$ is irrotational where $\overline{r} = x\ \mathbf{i} + y\ \mathbf{j} + z\ \mathbf{k}, r = |\overline{r}|$.

9. Show that $\nabla^2 f(r) = f''(r) + \frac{2}{r}f'(r)$,where $\overline{r} = x\ \mathbf{i} + y\ \mathbf{j} + z\ \mathbf{k}$.

10. Show that div$(r^n\overline{r}) = (n + 3)r^n$,where $\overline{r} = x\ \mathbf{i} + y\ \mathbf{j} + z\ \mathbf{k}$.

11. Show that the vector field $\overline{V} = (\sin y + z)\mathbf{i} + (x \cos y - z)\mathbf{j} + (x - y)\mathbf{k}$ is irrotational.

12. Evaluate $\int_c \overline{f} \cdot d\overline{r}$, where c is the part of the spiral $\overline{r} = (a \cos\theta, a \sin\theta, a\theta)$ corresponding to $0 \leq \theta \leq \frac{\pi}{2}$ and $\overline{f} = r^2\mathbf{i}, |\overline{r}| = r$.

13. Evaluate $\int_A^B[2xy\ dx + (x^2 - y^2)\ dy]$ along the arc of the circle $x^2 + y^2 = 1$ in the first quadrant from $A(1, 0)$ to $B(0, 1)$.

14. Show that $\phi = (e^x \cos y + yz)\mathbf{i} + (xz - e^x \sin y)\mathbf{j} + (xy + z)\mathbf{k}$ is conservative and find a scalar potential for it.

15. Apply Green's theorem in the plane to evaluate $\int_c[(y - \sin x)\ dx + \cos x\ dy]$ where c is the triangle enclosed by the lines $y = 0, x = \pi, \pi y = 2x$.

16. Evaluate by Green's theorem $\int_c(x^2 - \cosh y)dx + (y + \sin x)dy$, where c is the rectangle with vertices $(0, 0), (\pi, 0), (\pi, 1), (0, 1)$.

17. Using divergence theorem evaluate $\int\int_S xyz\ dydz$ where S is surface of the parallelepiped $0 \leq x \leq 3, 0 \leq y \leq 2, 0 \leq z \leq 1$.

18. Using divergence theorem evaluate $\int\int_S (e^x \, dydz - ye^x \, dzdx + 3z \, dxdy)$ where S is the surface of the cylinder $x^2 + y^2 \leq c^2, 0 \leq z \leq h$.

19. Using Stoke's theorem evaluate $\int_c \sin z \, dx - \cos x \, dy + \sin y \, dz$ where c is the boundary of the rectangle $0 \leq x \leq \pi, 0 \leq y \leq 1$ and $z = 3$.

20. By line integral evaluate the area of the loop of wiscrete's folium $x^3 + y^3 = 3axy$.

21. Find the average value of $f(x, y) = x \cos xy$ over rectangle $R : 0 \leq x \leq \pi, 0 \leq y \leq 1$.

22. Find the flux of $\overline{F} = yz\mathbf{j} + z^2\mathbf{k}$ outward through the surface S cut from the cylinder $y^2 + z^2 = 1, z \geq 0$ by the planes $x = 0 \& x = 1$.

23. Find the flux of $\overline{F} = y\mathbf{i} + x\mathbf{j} - z^2\mathbf{k}$ outward through the parabolic cylinder $y = x^2, 0 \leq x \leq 1, 0 \leq z \leq 4$.

24. Find the value of $\int\int_E e^{\frac{y}{x}} dS$ if the domain E of integration is the triangle bounded by the straight lines $y = x, y = 0 \; \& x = 1$.

25. Apply Stoke's theorem to evaluate $\int_c (y \; dx + z \; dy + x \; dz)$ where c is the curve of intersection of $x^2 + y^2 + z^2 = a^2 \; \& x + z = a$.

26. Evaluate $\int\int_S (a^2x^2 + b^2y^2 + c^2z^2)^{\frac{-1}{2}} \; dS$, where S is the surface of the ellipsoid $ax^2 + by^2 + cz^2 = 1$.

27. Evaluate $\int\int_S y \; dS$ where S is the surface $z = x + y^2, 0 \leq x \leq 1, 0 \leq y \leq 1$.

28. Evaluate $\int\int_S z \; dS$ where S is the surface whose sides S_1 are given by the cylinder $x^2 + y^2 = 1$ whose bottom S_2 is the disk $x^2 + y^2 \leq 1$ in the plane $z = 0$, and whose top S_3 is the part of the plane $z = 1 + x$ that lies above S_2.

29. Find the flux of the vector field $\overline{F}(x, y, z) = z \; \mathbf{i} + y \; \mathbf{j} + x \; \mathbf{k}$ across the unit sphere $x^2 + y^2 + z^2 = 1$.

30. Evaluate $\int\int_S \overline{F} \; dS$ where $\overline{F}(x, y, z) = y \; \mathbf{i} + x \; \mathbf{j} + z \; \mathbf{k}$ and S is the boundary of the solid region E enclosed by the paraboloid $z = 1 - x^2 - y^2$ and the plane $z = 0$.

31. Use Stoke's theorem to evaluate $\int_c \overline{F} \cdot d\overline{r}$ where $\overline{F}(x, y, z) = -y^2 \; \mathbf{i} + x \; \mathbf{j} + z^2 \; \mathbf{k}$ and c is the curve of intersection of the plane $y + z = 2$ and the cylinder $x^2 + y^2 = 1$.

32. Use Stoke's theorem to compute the integral $\int\int_S curl \overline{F} \cdot dS$, where $\overline{F}(x, y, z) = xz \; \mathbf{i} + yz \; \mathbf{j} + xy \; \mathbf{k}$ and S is the part of the sphere $x^2 + y^2 + z^2 = 4$ that lies inside the cylinder $x^2 + y^2 = 1$ and above the xy-plane.

33. Using divergence theorem find the flux of the vector field $\overline{F}(x, y, z) = z \; \mathbf{i} + y \; \mathbf{j} + x \; \mathbf{k}$ over the unit sphere $x^2 + y^2 + z^2 = 1$.

34. Evaluate $\int\int_S \overline{F} \cdot \mathbf{n} \, dS$ where $\overline{F}(x, y, z) = xy \; \mathbf{i} + (y^2 + e^{xz^2}) \; \mathbf{j} + \sin xy \; \mathbf{k}$ where S is the surface of the region E bounded by the parabolic cylinder $z = 1 - x^2$ and the planes $z = 0, y = 0, y + z = 2$.