Visvesvaraya National Institute of Technology, Nagpur
Department of Mathematics

Assignment (ODE-3) **Subject: MAL 102**

1. Give an example to show that the product of two solutions of a DE of the form $y\prime\prime + P(x)y\prime + Q(x)y = 0$ need not be a solution.
2. Show that $y_1(x) = 3e^{2x} - 1$ and $y_2(x) = e^{-x} + 2$ are solutions of the differential equation $yy\prime\prime + 2y\prime - (y\prime)^2 = 0$, but neither $2y_1$not $y_1 + y_2$ is a solution.
3. Find the general solution of the following differential equations:
   (i) $y'' + y' \tan x = \cos x$. (ii) $y''' - 6y'' + 11y' - 6y = e^x$.
   (iii) $y'^v + 36y'' - 4y - 6y = \cos^2 x - \cosh x$. (iv) $(D^3 + 2D^2 + D)y = e^{2x} + x^2 + x + \sin 2x$
4. Solve the following initial value problems:
   (i) $y'' + 4y = 8\sin 2x$ with $y(0) = 6, y'(0) = 8$.
   (ii) $y'' + -2y' + y = 2xe^{2x} + 6e^x$ with $y(0) = 1, y'(0) = 0$.
   (iii) $y'' + 5y' + 4y = 16x + 20e^x$ with $y(0) = 0, y'(0) = 3$.
   (iii) $y'' - y = 3x^2$ with $y(0) = 1, y'(0) = 2$.
5. Use method of variation of parameters to find the general solution of the following differential equations:
   (i) $y''+y = \frac{1}{1+\sin x}$ (ii) $y''+y = \sec x \csc x$ (iii) $y''-2y'+y = e^x \sin^{-1} x$ (iv) $(D^2+1)y = \csc x$
   (v) $(D^2 + 4)y = \tan 2x$ (vi) $(D^2 - 4D + 4)y = \frac{e^{2x}}{x^2}$ (vii) $(D^2 - D - 2)y = 3e^{(e^x+3x)}$
   (ix)$(D^2 - 1)y = \frac{2}{1+e^x}$ (x) $(D^2 + 2D + 1)y = e^{-x\log x}$ (viii) $(D^2 - 3D + 2)y = \cos e^{-x}$.
6. Find particular integral of the following differential equations with the given complementary functions:
   (i) $(x^2 + 1)y'' - 2xy' + 2y = 6(x^2 + 1)^2$ with $y_c = c_1x + c_2(x^2 - 1)$.
   (ii) $(x^2 + 2x)y'' - 2(x + 1)y' + 2y = (x + 2)^2$ with $y_c = c_1(x + 1) + c_2x^2$.
   (iii) $(x + 1)^2y'' - 2(x + 1)y' + 2y = 1$ with $y_c = c_1(x + 1) + c_2(x + 1)^2$.
   (iv) $x^2y'' - x(x + 2)y' + (x + 2)y = x^3$ with $y_c = c_1x + c_2xe^x$.
   (v) $\sin^2 x\ y'' - \sin 2x\ y' + (\cos^2 x + 1)y = \sin^3 x$ with $y_c = c_1 \sin x + c_2x \sin x$.
7. Find the general solution of the following differential equations:
   (i) $x^3\frac{d^4y}{dx^4} + 8x^2\frac{d^3y}{dx^3} + 8x\frac{d^2y}{dx^2} - 8\frac{dy}{dx} = 0$ (ii) $x^2\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} - 4y = x^2 + 2\log x$
   (iii) $x^4\frac{d^3y}{dx^3} + 2x^3\frac{d^2y}{dx^2} + -x^2\frac{dy}{dx} + xy = 1$ (iv) $x^2\frac{d^2y}{dx^2} + 3x\frac{dy}{dx} + y = \frac{1}{x}\log\log x$
   (v) $(x+1)^2\frac{d^2y}{dx^2} - 3(x+1)\frac{dy}{dx} + 4y = (x+1)^2$ (vi)$(5x+1)^2\frac{d^2y}{dx^2} - 4(5x+1)\frac{dy}{dx} + 35y = (5x+1)^3$
   (vii) $x^2\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = \frac{x}{x+1}$ (viii) $x^2\frac{d^2y}{dx^2} + 3x\frac{dy}{dx} + 5y = \frac{1}{x}\sec\log x^2$
8. Solve the following system of differential equations:
   (i) $\frac{dx}{dt} + \frac{dy}{dt} - 2x - 4y = e^t$; $\frac{dx}{dt} + \frac{dy}{dt} - y = e^{4t}$ (ii) $2\frac{dx}{dt} + \frac{dy}{dt} - x - y = e^{-t}$; $\frac{dx}{dt} + \frac{dy}{dt} + 2x + y = e^t$
   (iii) $\frac{dx}{dt} + \frac{dy}{dt} + 2y = \sin t$; $\frac{dx}{dt} + \frac{dy}{dt} - x - y = 0$ (v) $(D^2 - 2)x - 3y = e^{2t}$; $(D^2 + 2)y + x = 0$
   (vi) $(D^2 + 4)x + y = te^{3t}$; $(D^2 + 1)y - 2x = \cos^2 t$
   (iv) $2\frac{dx}{dt} + \frac{dy}{dt} + x + y = t^2 + 4t$;$\frac{dx}{dt} + \frac{dy}{dt} + 2x + 2y = 2t^2 - 2t$
9. Show that the particular integral of the differential equation $y'' + y = f(x)$ is

$$y_p(x) = \int f(t)\sin(x - t)dt.$$

   Also find a similar formula for particular integral of the differential equation $y'' + k^2y = f(x)$.