Visvesvaraya National Institute of Technology, Nagpur
Department of Mathematics

Assignment (ODE-2) **Subject: MAL-102**

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1. Find the orthogonal trajectories for the one parameter family of curves
   (i) $y = x - 1 + ce^{-x}$, (ii) $x = \dfrac{y^2}{4} + \dfrac{c}{y^2}$, (iii) $x^2 + y^2 + 2\lambda x = 2$ (iv) $x^{2/3} + y^{2/3} = a^{2/3}$
   (v) $y = cx^4$, (vi) $y = cx^n, n \in \mathbb{Z}^+$.

2. Find the orthogonal trajectories to the family of circles which are tangent to the y-axis at the origin.

3. Show that the following family of curves are self orthogonal: $(i)\, y^2 = 4c(x + c)$, where c is arbitrary parameter. (ii) $\dfrac{x^2}{c^2 + \lambda} + \dfrac{y^2}{d^2 + \lambda} = 1$, where c and d are fixed constants and λ is arbitrary parameter.

4. Find the orthogonal trajectories of the family of curves
   (i) $r = 2c \sin\theta$ (ii) $r = \dfrac{2a}{1 + \cos\theta}, a > 0$, (iii) $r = c(\sec\theta + \tan\theta)$,
   (iv) $r = a(1 - \cos\theta)$, (v) $r = a^\theta$ (vi) $r^n \sin\, n\theta = a^n$.

5. Find the oblique trajectories that intersect the following family of curves at the angle 45 degree:
   (i) $y = mx$ (ii) $x^2 + y^2 = a^2$.

6. Show that the initial value problem $xy\prime = 4y, y(0) = 1$ has no solution.
   What happens if we replace $y(0) = 1$ by $y(0) = 0$?
   Does this contradicts the existence theorem?

7. Discuss the applicability of the existence theorem to $xdy - ydx = 0, y(0) = 0$.

8. Test the existence and uniqueness of solutions of the following IVP's:
   (i) $y\prime = \dfrac{(1 + 2x + 3y)}{(2 + x^2 + y^2)}$, $y(0) = 0$ on the rectangle $R : \mid x \mid \leq 2, \mid y \mid \leq 1$.
   (ii) $y\prime = \dfrac{2xy}{(4 + x^2 + y^2)}$, $y(-1) = 1$ on the rectangle $R : \mid x + 1 \mid \leq 1, \mid y - 1 \mid \leq 2$.
   (iii) $y\prime = \dfrac{3xy}{(2 + \cos(xy))}$, $y(0) = 0$ on the rectangle $R : \mid x \mid \leq 2, \mid y \mid \leq 2$.
   (iv) $(x - 2y + 1)dy - (3x - 6y + 2)dx = 0, y(0) = 0$ on the rectangle $R : |x| \leq 1/4, |y| \leq 1/4$.
   (v) $xy' = 3y, y(1) = 1$ on the rectangle $|x - 1| \leq 2, |y - 1| \leq 4$.

9. Let $y\prime + P(x)y = Q(x)$, be a linear first order differential equation where $P(x), Q(x)$ are continuous in $\mid x - x_0 \mid \leq a$. Show that this problem has always a unique solution.

10. A first order IVP may have (i) unique solutions (or) (ii) more than one solution (or) (iii) no solution. Justify this statement.