Visvesvaraya National Institute of Technology, Nagpur
Department of Mathematics

Assignment(ODE-1) **Subject: MAL-102**

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1. Verify that all members of the family $y = \frac{1}{(x+c)}$ are solutions of the o.d.e $y' = -y^2$. Find a solution of the d.e. $y' = -y^2$ that is not a member of the above family.

2. Verify that all members of the family $y = cx + \frac{1}{c}$ are solutions of the d.e $y = xy' + \frac{1}{y'}$. Can you think of a solution of the d.e $y = xy' + \frac{1}{y'}$ that is not a member of the above family, if so, find it.

3. Find the governing o.d.e for the following family of curves: (i) $xy = Ae^x + Be^{-x}$ (ii) $y = k_1e^{2x} + k_2e^{-3x} + k_3e^x$.

4. Assume that $y = -x - \sqrt{4 - x^2}$ is an explicit solution of the IVP $(y + ax)y' + (ay + bx) = 0,\ y(0) = y_0$. Determine values for the constants a, b and y_0.

5. Reduce the following to first order o.d.e and hence solve : (i) $y'' + (1 + \frac{1}{y})(y')^2 = 0$ (ii) $xy'' + y' = 0$. (iii) $y'' + e^y(y')^3 = 0$, (iv)$yy'' = 2(y')^2$

6. Use appropriate substitution to solve the following o.d.e (i) $\dfrac{dy}{dx} = \sin(x + y) + \cos(x + y)$
   (ii) $x\dfrac{dy}{dx} = y(\log y - \log x + 1)$
   (iii)$(2 + 2x^2y^{1/2})ydx + (x^2y^{1/2} + 2)xdy = 0$
   (iv) $(1 - xy + x^2y^2)dx + (x^3y - x^2)dy = 0$
   (v)$\dfrac{xdx + ydy}{xdy - ydx} = \sqrt{\dfrac{a^2 - x^2 - y^2}{x^2 + y^2}}$

7. In each of the following determine the unknown constant A, such that the equation is exact and solve the resulting equation
   (i) $\left(\dfrac{1}{x^2} + \dfrac{1}{y^2}\right) dx + \left(\dfrac{Ax + 1}{y^3}\right) dy = 0$ (ii) $\left(\dfrac{Ay}{x^3} + \dfrac{y}{x^2}\right) dx + \left(\dfrac{1}{x^2} - \dfrac{1}{x}\right) dy = 0$

8. In each of the following equations determine the most general function $M(x, y)$ or $N(x, y)$ such that the equation is exact.
   (i) $(x^3 + xy^2)dx + N(x, y)dy = 0$ (ii) $M(x, y)dx + (2x^2y^3 + x^4y)dy = 0$

9. (a) Show that $\dfrac{1}{x^2 + y^2}$ is an integrating factor of a differential equation of the form $[y + xf(x^2 + y^2)]\,dx + [yf(x^2 + y^2) - x]\,dy = 0$. Hence solve the equation $[y + x(x^2 + y^2)^2]\,dx + [y(x^2 + y^2)^2 - x]\,dy = 0$.
   (b) Show that $\cos(x + y)$ is an integrating factor of a differential equation $ydx + (y + \tan(x + y))dy = 0$ and also find general solution.

10. Find all possible solutions of the following differential equations
    (i) $(xy)^{-1}dy - \dfrac{1}{x^2}dx = 0$ (ii) $(x - 4)y^4dx - x^3(y^2 - 3)dy = 0$

(iii) $(e^v+1)\cos u du + e^v(\sin u + 1)dv = 0$ (iv) $(y^2+2xy)dx - x^2dy = 0$(find an I.F. by multiplying the equation throughout by $y^n, n \in \mathbb{Z}$ and determine n so that the given equation is exact).

11. Show that
(i) $(Ax+By)dx + (Cx+Dy)dy = 0$ is exact if and only if $B = C$,
(ii)$(Ax^2+Bxy+Cy^2)dx + (Dx^2+Exy+Fy^2)dy = 0$ is exact iff $B = 2D$ and $E = 2C$.

12. Use the results in problem 10 to solve
(i) $(3x-y)dx - (x-y)dy = 0$ (ii) $(x^2+2y^2)dx + (4xy-y^2)dy - 0$.

13. Solve the following o.d.e:
(i) $(2xy^4e^y + 2xy^3 + y)dx + (x^2y^4e^y - x^2y^2 - 3x)dy = 0$ (ii) $(xy\text{sin}(xy) + \text{cos}(xy)ydx + (xy\text{sin}(xy) - \text{cos}(xy))xdy$

14. Solve the following initial value problems
(i) $8\cos^2 y dx + \csc^2 x dy = 0\,; y(\frac{\pi}{12}) = \frac{\pi}{4}$,
(ii) $(3x^2+9xy+5y^2)dx - (6x^2+4xy)dy = 0\,; y(2) = -6$.

15. Solve the following initial value problems
(i) $(x^2+1)\dfrac{dy}{dx} + 4xy = x\,; y(2) = 1$, (ii) $e^x(y - 3(e^x+1)^2)dx + (e^x+1)dy = 0\,; y(0) = 4$,
(iii) $\dfrac{dr}{d\theta} + r\tan\theta = \cos^2\theta\,; r(\dfrac{\pi}{4}) = 1$, (iv) $\dfrac{dx}{dt} - x = \sin x\,; x(0) = 0$.
(v) $y' + y\tan x = \sin 2x\,; y(0) = 1$

16. (a) Show that the d.e. $y' + P(x)y = Q(x)$(with $P(x) \neq 0$) is not exact.
(b) Let $\mu(x) = e^{\Delta p(x)dx}$. Show that the equation $\mu(x)(y' + P(x)y) = \mu(x)Q(x)$ is exact and hence solve it.

17. Solve the following o.d.e (i) $x\log x\dfrac{dy}{dx} + y = (\log x)^2$
(ii) $(1+x^2)y' + 2xy = x\text{sin}x$
(iii) $\dfrac{dx}{dy} = \frac{\text{sec}^2 y}{x^3 - 2x\text{tan}y}$

18. The initial value problem governing the current i, flowing in a series RL circuit when a sinusoidal voltage $v(t) = \text{sin}\omega(t)$ is applied, is given by(R, ω and L are constant) $iR + L\dfrac{di}{dt} = \text{sin}\omega t, \quad t \geq 0 \; i(0) = 0$. Find the current $i(t), \; t > 0$.

19. Solve the following d.e: (i) $y' + y = y^2$ (ii) $2xy' = 10x^3y^5 + y$(iii) $xy' + y = x^2y^2\text{ln}x$.

20. Find the integrating factor of the form x^py^q and solve $(4xy^2+6y)dx + (5x^2y+8x)dy = 0$.

21. Show that $F(x,y)$ is an integrating factor of the non-exact differential equation $M(x,y)dx + N(x,y)dy = 0$ if and only if $\left[N\dfrac{\partial F}{\partial x} - M\dfrac{\partial F}{\partial y}\right] = F\left[\dfrac{\partial M}{\partial y} - \dfrac{\partial N}{\partial x}\right]$.

22. Show that the reciprocal of any homogeneous function $P(x,y) = Ax^2 + 2Bxy + Cy^2$ (A, B, C are constants) is an integrating factor of $xdy - ydx = 0$.