**VISVESVARAYA NATIONAL INSTITUTE OF TECHNOLOGY, NAGPUR**
**DEPARTMENT OF MATHEMATICS**

**Topic: Assignment-3(PDE)** **MAL-201, B. Tech., III-Semester(2015).**

**Separation of Variables Method:**

1. Use separation of variables to find the solution of the following pdes:
(i) $4\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}=0$. (ii) $y\frac{\partial u}{\partial x}+x\frac{\partial u}{\partial y}=0$. (iii) $\frac{\partial^2 u}{\partial x^2}=\frac{\partial u}{\partial y}+2u$.
(iv) $u_{tt}=u_{xx}-au_t$, where $a\in(0,1)$ . (v) $u_{tt}+u_t+u=c^2u_{xx}$, $c>0$.

**Wave Equation:**

1. Solve the wave equation $u_{tt}=c^2u_{xx}, 0<x<l, t>0$ with the boundary conditions $u(0,t)=u(l,t)=0$ for $t>0$, initial conditions $u(x,0)=f(x)$ and $u_t(x,0)=g(x)$ for $0<x<l$.

2. Solve the above wave equation with the following conditions:
(i) $l=\pi$, $f(x)=x\cos(\frac{5x}{2})$ and $g(x)\equiv 0$. (ii) $l=4$, $f(x)\equiv 0$ and $g(x)=\begin{cases}x, & 0<x<2\\ 0, & 2<x<4\end{cases}$.
(iii) $l=\pi$, $f(x)\equiv 0$ and $g(x)=\sin x$. (iv) $c=4, l=2$, $f(x)=x(2-x)$ and $g(x)=1$.
(v) $c=5, l=\pi$, $f(x)=\sin 2x$ and $g(x)=\pi-x$.

3. Solve the wave equation $u_{tt}=c^2u_{xx}-\sin x, 0<x<\pi/2, t>0$ with the boundary conditions $u(0,t)=u(\pi/2,t)=0$ for $t>0$, initial conditions $u(x,0)=0$ and $u_t(x,0)=0$ for $0<x<\pi/2$.

4. Solve the wave equation $u_{tt}=9u_{xx}+\cos\pi x, 0<x<4, t>0$ with the boundary conditions $u(0,t)=u(4,t)=0$ for $t>0$, initial conditions $u(x,0)=x(4-x)$ and $u_t(x,0)=0$ for $0<x<4$.

5. Solve the initial boundary value problem$u_{tt}=c^2u_{xx}, 0<x<l, t>0$ with the boundary conditions $\frac{\partial u}{\partial x}(0,t)=\frac{\partial u}{\partial x}(l,t)=0$ for $t>0$, initial conditions $u(x,0)=x$ and $u_t(x,0)=g(x)$ for $0<x<l$.

6. Use D'Alembert solution to solve the initial value problem defining the vibrations of an infinitely long elastic string when (i) $f(x)=\sin 2x$, $g(x)=\cos 2x$. (ii) $g(x)\equiv 0, f(x)=kx(1-x)$.

**Heat Equation:**

1. Solve the heat equation $u_t=c^2u_{xx}, 0<x<l, t>0$ with the boundary conditions $u(0,t)=u(l,t)=0$ for $t>0$, initial condition $u(x,0)=f(x)$ for $0<x<l$.

2. Solve the above heat equation $u_t=3u_{xx}, 0<x<l, t>0$ with the boundary conditions $u(0,t)=u(l,t)=0$ for $t>0$, initial condition $u(x,0)=l\left(1-\cos\left(\frac{2\pi x}{l}\right)\right)$ for $0<x<l$.

3. Find the temperature distribution in a bar with insulated ends on the interval $(0,6)$ when the initial temperature is $e^{-x}$ for $0\leq x\leq 6$ and the thermal diffusivity $c^2$ is $4$.

4. A thin, homogeneous bar of length $L$ has initial temperature equal to $25^0C$, the right end $(x=L)$ is insulated while the left end $(x=0)$ is kept at zero temperature. Find the temperature distribution in the bar.

5. Solve the heat equation $u_t=c^2u_{xx}, 0<x<l, t>0$ with the boundary conditions $u(0,t)=1, u(l,t)=2$ for $t>0$, initial condition $u(x,0)=1$ for $0<x<l$.

6. An insulated rod of length $l$ has its ends $A$ and $B$ maintained at $0^0C$ and $100^0C$ respectively until steady state conditions prevail.
(i) If $B$ is suddenly reduced to $0^0C$ and maintained $0^0C$, find the temperature distribution in the bar at any time $t$.
(ii) If the temperature of $A$ is raised to $20^0C$ and reducing the temperature of $B$ to $80^0C$, find the temperature distribution in the bar at any time $t$.

7. A bar of 20cm long with insulted sides has its ends $A$ and $B$ maintained at temperatures $30^0C$ and $80^0C$ respectively, until steady-state conditions prevails. The temperature of $A$ is suddenly raised to $40^0C$ and at the same time that at $B$ is lowered to $60^0C$. Find the temperature distribution in the bar at time $t$.

**Laplace Equation:**

1. Solve for the steady state temperature distribution in a thin flat plate covering the rectangle $0 \leq x \leq 4, 0 \leq y \leq 1$, if the temperature on the vertical sides is zero while on the horizontal lower side it is $f(x) = \sin \pi x$ and on the horizontal upper side it is $g(x) = x(4 - x)$.

2. Solve the boundary value problem governing the steady state temperature distribution in a thin. flat rectangular plate given by $u_{xx} + u_{yy} = 0, 0 < x < a, 0 < y < b$ with the boundary conditions $u(0, y) = f(y), u(a, y) = g(y), 0 < y < b$ and $u(x, 0) = u(x, b) = 0, 0 < x < a$.

3. The boundaries of a thin rectangular plate are $x = 0, x = 1, y = 0, y = 2$. The steady state temperature distribution in the plate is to be determined when the vertical edges are insulated. The boundary value problem modelling the temperature distribution is, $u_{xx}+u_{yy} = 0, 0 < x < 1, 0 < y < 2$ with the boundary conditions $u_x(0, y) = u_x(1, y) = 0, 0 < y < 2, u(x, 0) = 0$ and $u(x, 2) = \begin{cases} x, & 0 < x < 0.5 \\ 1 - x, & 0.5 \leq x < 1 \end{cases}$.
Find the temperature distribution?

4. Solve the boundary value problem governing the steady state temperature distribution in a thin. flat rectangular plate given by $u_{xx} + u_{yy} = 0, 0 < x < a, 0 < y < b$ with the boundary conditions $u(0, y) = u(a, y) = 0, 0 < y < b$, $u_y(x, 0) = 0$and $u(x, b) = g(x), 0 < x < a$.

5. Solve the boundary value problem governing the steady state temperature distribution in a thin. flat rectangular plate given by $u_{xx} + u_{yy} = 0, 0 < x < 1, 0 < y < 1$ with the boundary conditions $u(0, y) = y, u_x(1, y) = -10, 0 < y < 1$ and $u(x, 0) = u(x, 1) = 0, 0 < x < 1$.

6. Obtain the steady state temperature distribution in a semi-circular plate of radius $a$ whose bounding diameter is kept at $0^0C$ and the temperature along the semi-circular boundary is given by
(i) $u(a, \theta) = 50^0C$ (ii) $u(a, \theta) = \begin{cases} 50\theta, & 0 < \theta < \frac{\pi}{2} \\ 50(\pi - \theta), & \frac{\pi}{2} \leq \theta < \pi \end{cases}$.