VISVESVARAYA NATIONAL INSTITUTE OF TECHNOLOGY, NAGPUR
DEPARTMENT OF MATHEMATICS
Topic: Assignment (Laplace Transforms) **MAL-201, B. Tech., III-Semester(2014).**

1. Find Laplace transform of the following functions:
(i) $\cos\sqrt{t}$ (ii) $\left(\sqrt{t} \pm \frac{1}{\sqrt{t}}\right)^3$ (iii) $\sin\sqrt{t}$ (iv)$\frac{t^{n-1}}{1-e^{-t}}$ (v) $e^{2t}\sin^4 t$
(vi) $\int_0^t \frac{1-e^{-u}}{u}\,du$ (vii) $\int_0^t \frac{\sin u}{u}\,du$ (viii)$\int_0^t ue^{-u}\sin 4u\,du$ (ix) $\int_0^t \frac{e^{-4u}\sin 3u}{u}\,du$

2. If $L\{f(t)\} = \frac{e^{-\frac{1}{s}}}{s}$, find $L\{e^{-t}f(3t)\}$.

3. Given that $L\{\sin\sqrt{t}\} = \frac{\sqrt{\pi}}{2s^{\frac{3}{2}}}e^{-\frac{1}{4s}}$, prove that $L\left\{\frac{\cos\sqrt{t}}{\sqrt{t}}\right\} = \sqrt{\frac{\pi}{s}}e^{-\frac{1}{4s}}$.

4. Evaluate the following integrals using Laplace transforms:
(i) $\int_0^\infty t^2e^{-4t}\sin 2t\,dt$ (ii) $\int_0^\infty e^{-2t}\sin^3 t\,dt$ (iii) $\int_0^\infty \frac{\cos 6t-\cos 4t}{t}\,dt$
(iv) $\int_0^\infty \frac{e^{-t}\sin^2 t}{t}\,dt$ (v) $\int_0^\infty \frac{e^{-2t}(2\sin t-2\sinh t)}{t}\,dt$ (vi) $\int_0^\infty\int_0^t \frac{e^{-t}\sin u}{u}\,dudt = \frac{\pi}{4}$.

5. Find the Laplace Transform by expressing $f(t)$ in unit step functions
(i) $f(t) = \begin{cases} 2, & 0 < t < \pi, \\ 0, & \pi < t < 2\pi \\ \sin t, & t \geq 2\pi \end{cases}$ (ii) Staircase function $f(t) = \begin{cases} 1, & 0 < t < 1, \\ 2, & 1 < t < 2\pi \\ 3, & 2 < t < 3 \\ \ldots\ldots\ldots \end{cases}$

6. Find the Laplace transform of the following periodic functions:
(i) Half wave rectifier (ii) Sawtooth wave (iii) Full wave rectification of $|\sin\omega t|$
(iv) and$f(t) = \begin{cases} 1, & 0 \leq t \leq 2 \\ -1, & 2 \leq t \leq 4, \end{cases}$ and $f(t+4) = f(t)$ for all $t \geq 0$
(v) $f(t) = \begin{cases} \cos t, & 0 \leq t \leq \pi \\ -1, & \pi \leq t \leq 2\pi, \end{cases}$ and $f(t+2\pi) = f(t)$ for all $t \geq 0$.

7. Find inverse Laplace transform of the following functions:
(i) $\frac{1}{s}e^{-\frac{1}{\sqrt{s}}}$ (ii) $\frac{1}{s}e^{-\frac{1}{s}}$ (iii) $\frac{1}{s}\sin\frac{1}{s}$ (iv) $\frac{64}{81s^4-256}$ (v) $\frac{1}{\sqrt[3]{8s-27}}$ (vi) $\frac{s+1}{(s^2+2s+2)^2}$
(vii) $\cot^{-1}\left(\frac{s+a}{b}\right)$ (viii) $\tan^{-1}\left(\frac{2}{s^2}\right)$ (ix)$\int_s^\infty \left(\frac{u}{u^2+a^2} - \frac{u}{u^2+b^2}\right)du$ (x) $\int_s^\infty \left(\tan^{-1}\frac{2}{u^2}\right)du$
(xi)$\frac{1}{(s^2+a^2)^2}$ (xii) $\frac{1}{s^3(s+1)}$ (xiii) $\frac{e^{-s}}{\sqrt{s+1}}$ (xiv) $\frac{2+5s}{s^2e^{4s}}$ (xv) $\frac{1}{s^2-e^{-as}}$ (xvi) $\frac{e^{-3s}}{(s-4)^2}$
(xvii) $\frac{1}{s+e^{-s}}$ (xviii) $\frac{s^3+6s^2+14s}{(s+2)^4}$ (xix) $\frac{s^2+2s+3}{(s^2+2s+2)(s^2+2s+5)}$ (xx)$\frac{s^3-3s^2+6s-4}{(s^2-2s+2)^2}$

8. If $L^{-1}\left\{\frac{1}{\sqrt{s}}e^{-\frac{1}{s}}\right\} = \frac{\cos 2\sqrt{t}}{\sqrt{\pi t}}$, show that $L^{-1}\left\{\frac{1}{\sqrt{s}}e^{-\frac{a}{s}}\right\} = \frac{\cos 2\sqrt{at}}{\sqrt{\pi t}},\ a > 0.$

9. If $L^{-1}\left\{\frac{s^2-1}{(s^2+1)^2}\right\} = t\cos t$, show that $L^{-1}\left\{\frac{9s^2-1}{(9s^2+1)^2}\right\} = \frac{t}{9}\cos\frac{t}{3}$.

10. Use convolution theorem to find inverse Laplace transform of the following functions:
(i) $\frac{16}{(s-2)(s+2)}$ (ii) $\frac{1}{s(s+1)(s+2)}$ (iii) $\frac{1}{(s^2+4)(s+1)^2}$

11. Solve the following differential equations by using Laplace transforms:
(i) $y'' + 2y' + 5y = e^{-t}\sin t; y(0) = 0, y'(0) = 1$
(ii) $y''' - 3y'' + 3y' - y = t^2e^t; y(0) = 1, y'(0) = 0, y''(0) = -2$

(iii)$y' - 4y + 3\int_0^t y(\tau)d\tau = 1; y(0) = 1$
(iv) $y' - y - 6\int_0^t y(\tau)d\tau = \sin t; y(0) = 2$
(v) $y' + y = f(t), y(0) = 2$ where $f(t) = \begin{cases} 0, & 0 \le t \le \frac{\pi}{2} \\ \cos t, & t \ge \frac{\pi}{2} \end{cases}$
(vi) $y'' + 2y' + 5y = \delta(t-2), y(0) = 0, y'(0) = 0$
(vii) $y'' + 2y' + 10y = 6\delta(t-2) - 3\delta(t-3), y(0) = 0, y'(0) = 0$
(viii)$ty'' + 2ty' + 2y = 2, y(0) = 1, y'(0) = c$,
(ix) $y'' + 3y' + 2y = e^{-t}, y(0) = 0, y'(0) = -1.$

12. Using convolution theorem, solve
(i) $f(t) + \int_0^t f(\tau)\cos(t-\tau)d\tau = e^{-t}$ (ii) $f(t) = \cos t + e^{-t}\int_0^t f(\tau)e^{\tau}d\tau$.

13. Find the solution of the following IVPs:
(i) $y'' + 8y' + 17y = f(t), y(0) = 0, y'(0) = 0$,
where $f(t)$ is periodic function $f(t) = \begin{cases} 1, 0 \le t \le \pi \\ 0, \pi < t < 2\pi; \end{cases}$ and $f(t+2\pi) = f(t)$ for all $t \ge 0$
(ii) $y'' + 9y = f(t), y(0) = 1, y'(0) = 0,$
where $f(t)$ is periodic function $f(t) = \begin{cases} \cos t, 0 \le t \le \pi \\ 0, \pi < t < 2\pi; \end{cases}$ and $f(t+2\pi) = f(t)$ for all $t \ge 0$
(iii) $y'' + 9y = f(t), y(0) = 1, y'(0) = 0,$
where $f(t)$ is periodic function $f(t) = \begin{cases} 0, 0 \le t \le \pi \\ \sin t, \pi < t < 2\pi; \end{cases}$ and $f(t+2\pi) = f(t)$ for all $t \ge 0$
(iv) Find solution of the integro differential equation $y' + 5y + 4\int_0^t y(\tau)d\tau = f(t), y(0) = 2,$
where $f(t)$ is a rectangular pulse given by $f(t) = \begin{cases} 0, & 0 \le t < a \\ k, & a \le t \le b \\ 0, & t > b \end{cases}$.

14. The differential equation governing the flow of current $i(t)$ in an LR series circuit is given by $L\dfrac{di}{dt} + iR = E(t)$. Find the current $i(t)$ if the current is initially zero and
(i)$E(t) = \begin{cases} 0, 0 \le t \le 2 \\ 5, t \ge 2 \end{cases}$ (ii)$E(t) = \begin{cases} \sin t, 0 \le t < \frac{\pi}{2} \\ 0, t \ge \frac{\pi}{2} \end{cases}$

15. Prove that,
(i) Laplace transform of the function e^{t^2} does not exist
(ii) $\cos s$ can't be Laplace transform of any function.

16. Let $f(t)$ be a periodic function on $[0, \infty)$ with fundamental period T.
Show that $\int_{nT}^{(n+1)T} e^{-st}f(t)dt = e^{-nsT}\int_0^T e^{-st}f(t)dt$ for all $n \in \mathbb{N}$.