

VISVESVARAYA NATIONAL INSTITUTE OF TECHNOLOGY, NAGPUR
DEPARTMENT OF MATHEMATICS

Assignment on Z-Transforms

MAL-201, B.Tech., III-Semester(2014).

Z-Transforms:

1. Find the Z-transform of following sequence $\{f_n\}$, where f_n is given by

(i) $\frac{e^{-2n}}{n!}$ (ii) $e^{in\theta}$ (iii) $n^2 e^n$ (iv) $a^{-n} \cos \beta n$ (v) $\frac{(n+1)}{n!} a^n$ (vi) n^4

2. Let $Z\{f_n\} = F(z)$. Find f_0, f_1, f_2 where $F(z)$ is given by

(i) $\frac{z^2}{z^2+1}$ (ii) $\frac{z^3+5z^2+3z-1}{(z-1)^3(z+2)}$ (iii) $\frac{3z^2-4z+7}{(z-1)^3}$

3. (i) If $Z\{f_n\} = \frac{z^2-3z+5}{(z-1)(z+2)}$, then find $\lim_{n \rightarrow \infty} f_n$?

(ii) If $Z\{f_n\} = \frac{3z^3+5z^2-7z+1}{(z+2)^2(z-1)}$, then find $\lim_{n \rightarrow \infty} f_n$?

4. When can we say that a sequence $\{f_n\}$ is of exponential type?

State existence theorem of Z-transforms.

5. Show that $\left\{ \frac{1}{n!} \right\} * \left\{ \frac{1}{n!} \right\} = \left\{ \frac{2^n}{n!} \right\}$.

6. Verify convolution theorem for the following sequences:

(i) $f_n = 1$ and $g_n = n(3)^n$ (ii) $f_n = n$ and $g_n = n^2$ (iii) $f_n = e^n$ and $g_n = e^n$.

7. Find the inverse Z-transform for the following functions:

(i) $\frac{7z-11z^2}{(z-1)(z-2)(z+3)}$ (ii) $\frac{z^3+2z^2+29z}{(z-1)(z+3)^2}$ (iii) $\frac{z^2}{(z-1)(z-3)}$

(iv) $\frac{6z^2}{(z-1)^4(z+5)}$ (v) $\frac{z}{(z-1)^4}$

8. Solve the following difference equations using Z-transforms

(i) $y_{n+2} - 3y_{n+1} + 2y_n = 0, y_0 = -1, y_1 = 2.$

(ii) $y_{n+2} + 5y_{n+1} + 4y_n = 2^n, y_0 = 1, y_1 = -4.$

(iii) $y_{n+3} - 6y_{n+2} + 12y_{n+1} - 8y_n = 1, y_0 = 1, y_1 = 1, y_2 = 2.$

(iv) $y_{n+1} - 3y_n = 0, y_0 = 1.$

(v) $y_{n+3} + 2y_{n+2} - y_{n+1} - 2y_n = 1, y_0 = 1, y_1 = -1, y_2 = 3.$