Visvesvaraya National Institute of Technology, Nagpur
Department of Mathematics

Assignment II

Course: Numerical Analysis (MAL-202) **Branch: Civil/Mining Engg.**

1. Evaluate $\int_0^2 e^x\,dx$ using the Simpson's rule with $h = 1$ and ½.. Find a bound on the error in each case. Compare with the exact solution.
2. Evaluate the integrals (i) $I = \int_0^2 \frac{dx}{3+4x}$ (ii) $I = \int_0^2 \frac{dx}{x^2+2x+10}$
   (i) By Gauss-Legendre two-point and three-point formula.
   (ii) Write I as I_1+I_2 where $I_1=\int_0^1 f(x)dx$ and $I_2=\int_1^2 f(x)dx$. Then evaluate each of the integrals by Gauss –Legendre two-point and three-point formula. Compare with the exact solution.
3. Evaluate the integrals $\int_0^2 \frac{dx}{5+3x}$ using the open type formula
   $\int_{x_0}^{x_3} f(x)\,dx = \frac{3h}{2}[f(x_1) + f(x_2)] + \frac{3h^3}{4} f''(\xi) \quad x_0 < \xi < x_3$
   Where $x_i = x_0 + ih$, i=0(1)3. Find a bound on the error. Compare with the exact solution.
4. Evaluate the integral $I=\int_{-1}^1 (1-x^2)^{3/2} \cos x\,dx$ using the Gauss-Legendre 3 point formula.
5. Compute $\int_{\pi/4}^{\pi/2} \frac{\cos x \log(\sin x)dx}{\sin^2 x+1}$ correct up to 3 decimal places.
   (a) Using trapezoidal rule and Romberg integration.
   (b) Using Simpson's rule and Romberg integration.
6. Evaluate the integrals $I = \int_0^1 \frac{dx}{1+x}$ using (i) composite trapezoidal rule (ii) composite Simpson's rule with 2, 4 and 8 equal subintervals.
7. Calculate $\int_0^{1/2} \frac{x\,dx}{\sin x}$
   (i) Using the open type formulas
   (ii) Using the semi-open type formulas
   (iii) Using the trapezoidal rule with h=1/2, 1/4, 1/8 and Romberg integration. Assume $f(0)$ is taken as the limiting value.
8. Derive Gauss-Chebyshev one point, two point and three point formula for integration.
9. Find the remainder of the Simpson's three-eight rule
   $\int_{x_0}^{x_3} f(x)dx = \frac{3h}{8}[f(x_0) + 3f(x_1) + f(x_2) + f(x_3)]$
   For equally spaced points $x_i = x_0 + ih$, i=1, 2, 3. Use this rule to approximate the value of the integral $I = \int_0^1 \frac{dx}{1+x}$. Also, find a bound on the error.
10. Find the quadrature formula

   $\int_0^1 f(x)\frac{dx}{\sqrt{x(1-x)}} = \alpha_1 f(0) + \alpha_2 f(\tfrac{1}{2}) + \alpha_3 f(1)$

   Which is exact for polynomials of highest possible degree? Then use the formula on $\int_0^1 \frac{dx}{\sqrt{x-x^3}}$ and compare with the exact value.