DEPARTMENT OF MATHEMATICS

Course Book for
M.Sc. (Mathematics)

Visvesvaraya National Institute of Technology, Nagpur

October 2015

Brief about M.Sc. Programs:

Department of Mathematics offer M. Sc. program, namely, *M. Sc. in Mathematics.* The objective of the M. Sc. (Mathematics) is to develop highly qualified/trained mathematicians to cater to the needs of the industry, teaching and research institutions. Department of Mathematics has highly qualified, motivated, dynamic and experienced faculty members. The M.Sc. (Mathematics) programme intended to offer a balanced combination of core and applied courses of Mathematics. It also emphasizes advanced developments in the field of analysis, fluid mechanics, mathematical physics and scientific computing. This M.Sc. programme is designed for four semesters. In the first semester a course of Introduction to Computer Programming and another course of communication skill are included which will be helpful to students in their professional career. In addition to the regular courses, three electives courses in third and fourth semesters are available for the students. There is provision for each student to give a seminar in 3rd semester and complete a dissertation in fourth semester under the guidance of highly qualified faculty members. In these activities, student explores a specific topic, surveys the available literature and submits a critical review in the form of a report which may also include original theoretical results and/or results of experimental work. This process provides an initiation into mathematical research and also equips the student with the skills of presentation of research/technical report. This will be useful in developing awareness, aspiration and innovative ability to solve new scientific problems.

CREDIT REQUIREMENT FOR POST GRADUTE STUDIES

POST GRADUTE CORE (UC)		**POST GRADUTE ELECTIVE (UE)**	
Category	Credit	Category	Credit
Core	54	Electives (DE)	9
Grand Total UC + UE			63

The number of credits attached to a subject depends on number of classes in a week. For example a subject with 3–1–0(L–T–P) means it has 3 Lectures, 1 Tutorial and 0 Practical in a week. This subject will have four credits (3×1+1×1+0×1=4). If a student is declared pass in a subject, then he/she gets the credits associated with that subject. Depending on marks scored in a subject, student is given a Grade. Each grade has got certain grade points as follows:

Grades	AA	AB	BB	BC	CC	CD	DD	FF
Grade Points	10	09	08	07	06	05	04	Fail

The performance of a student will be evaluated in terms of two indices, viz., the Semester Grade Point Average (SGPA) which is the Grade Average for a semester and Cumulative Grade Point Average (CGPA) which is the Grade Point Average for all the completed semesters at any point in time. SGPA & CGPA are:

$$SGPA = \frac{\sum_{Semester} (Course\ credits\ X\ Grade\ points)\ for\ all\ courses\ except\ audit}{\sum_{Semester} (Course\ credits)\ for\ all\ courses\ except\ audit}$$

$$CGPA = \frac{\sum_{All\ Semester} (Course\ credits\ X\ Grade\ points)\ for\ all\ courses\ with\ pass\ grade\ except\ audit}{\sum_{All\ Semester} (Course\ credits)\ for\ all\ courses\ except\ audit}$$

Students can Audit a few subjects, i.e., they can attend the classes and do home work and give exam also, but they will not get any credit for that subject. Audit subjects are for self enhancement of students.

Details about Faculty members of Mathematics Department

S.No.	Name of faculty Member	Designation	Qualification	Area of Specialization
1	Dr. G. P. Singh	Professor and Head	Ph.D.	Relativity and Cosmology, Mathematical Modelling
2	Dr. P. P. Chakravarthy	Associate Professor	Ph.D.	Numerical Analysis, Numerical Analysis of Singular Perturbation problem
3	Dr. Pallavi Mahale	Assistant Professor	Ph.D.	Functional Analysis and Operator equations
4	Dr. R. P. Pant	Assistant Professor	Ph.D.	Functional Analysis, General Topology, Fixed point theory
5	Dr. G. Naga Raju	Assistant Professor	Ph.D.	Partial Differential Equations, Spectral Element Methods, Parallel Computing.
6	Dr. M. Devakar	Assistant Professor	Ph.D.	Fluid Dynamics
7	Dr. V. V. Awasthi	Assistant Professor	Ph.D.	Algebraic Topology and its Application
8	Dr. Pradip Roul	Assistant Professor	Ph.D.	Numerical Analysis, ODE, Fractional Calculus
9	Dr. Deepesh Kumar Patel	Assistant Professor	Ph.D.	Fixed Point Theory
10	Dr. Jyoti Singh	Assistant Professor	Ph.D.	Commutative Algebra
11	Dr. A. Satish Kumar	Assistant Professor	Ph.D.	Approximation Theory

Scheme of Instruction for M.Sc. (Mathematics)

I Semester				II Semester			
Code	**Course**	**L-T-P**	**Cr**	**Code**	**Course**	**L-T-P**	**Cr**
MAL511	Linear Algebra	3-0-0	3	MAL521	Complex Analysis	3-0-0	3
MAL512	Real Analysis	3-0-0	3	MAL522	Topology	3-0-0	3
MAL513	Theory of Ordinary Differential Equations	3-0-0	3	MAL523	Algebra	3-0-0	3
MAL514	Discrete Mathematics	3-0-0	3	MAL524	Partial Differential Equation	3-0-0	3
CSL501	Introduction to Computer Programming	3-0-0	3	MAL525	Numerical Analysis	3-0-0	3
HUL505	Communication Skill	Audit course	0	MAL526	Numerical Computation laboratory	0-0-2	1
		Total	**15**			**Total**	**16**
III Semester				**IV Semester**			
MAL531	Functional Analysis	3-0-0	3	MAL541	Measure Theory and Integration	3-0-0	3
MAL532	Operations Research	3-0-0	3	MAL542	Integral Transform and Integral Equations	3-0-0	3
MAL533	Fluid Dynamics	3-0-0	3	MAD502	Project Phase II	4-0-0	4
MAL534	Probability & Statistics	3-0-0	3				
MAD501	Project Phase I	1-0-0	1				
ELECTIVE (Any one)				**ELECTIVE (Any Two)**			
MAL535	Relativity	3-0-0	3	MAL543	Operator Theory	3-0-0	3
MAL536	Numerical Solutions of Differential Equations	3-0-0	3	MAL544	Finite Element Methods	3-0-0	3
MAL537	Mathematical Modelling	3-0-0	3	MAL545	Numerical methods for Hyperbolic problems	3-0-0	3
				MAL546	Bio Mechanics	3-0-0	3
				MAL547	Multivariate Data Analysis	3-0-0	3
				MAL548	Financial Mathematics	3-0-0	3
				MAL549	Nonlinear Dynamical Systems	3-0-0	3
				MAL550	Numerical Linear Algebra	3-0-0	3
		Total	**16**			**Total**	**16**

Scheme of Instruction for Ph. D. (Mathematics)

MAL601	Singular Perturbation Theory	3-0-0	3	MAL602	Fitted Numerical Methods for Singular Perturbation Problems	3-0-0	3
MAL603	Advanced Fluid Dynamics	3-0-0	3	MAL604	Matrix Iterative Analysis	3-0-0	3
MAL605	Theory of Micropolar and Couplestress fluids	3-0-0	3	MAL606	Cosmology	3-0-0	3
MAL607	Fixed Point Theory and Applications	3-0-0	3				

Syllabus

Note: For all text books/reference books, the latest editions should be used.

MAL 511 - Linear Algebra **[(3-0-0); Credit: 3]**

Objective: The objective of this subject is to expose student to understand the importance of Linear Algebra to improve ability to think logically, analytically, and abstractly.

Vector spaces over fields, Subspaces, Bases and dimension.Matrices, Rank, System of linear equations, Guass elimination.
Linear transformations, Representation of linear transformation by matrices, Rank-nullity theorem, Linearfunctionals, Annihilator, Double dual, Transpose of a linear transformation.
Characteristic values and characteristic vectors of linear transformations, Diagonalizability, Minimal polynomial of a linear transformation, Cayley Hamilton theorem, Invariant subspaces.
Direct sum decompositions, Invariant direct sums, The primary decomposition and the Rational form, The Jordan form. Inner product spaces, Orthonormal basis, Gram-Schmidt Theorem.

Text books:

1. K. Hoffman and R. Kunze, Linear Algebra, Pearson Education (India), 2003.
2. M. Artin: Algebra, Prentice Hall of India, 2005.

Reference Books:

1. I. N Herstein: Topics in Algebra, 2 nd Edition, John-Wiley, 1999.
2. S. Lang: Linear Algebra, Springer Undergraduate Texts in Mathematics, 1989.
3. S. Kumeresan: Linear Algebra: A Geometric Approach, Prentice Hall of India, 2004.

MAL512 - Real Analysis **[(3-0-0); Credit: 3]**

Objective: The main objective of this course is to express the basic concepts of Real Analysis. Further, students will be exposed to continuity, differentiability, integrability of real functions, convergence of sequence, series and functions (uniform & point wise).

Real number system and its structure, Infimum, Supremum, Dedekind cuts. (Proofs omitted)

Functions of single real variable, Limits of functions, Continuity of functions, Types of discontinuities.Uniform continuity. Differentiability and Mean Value Theorems. Metric spaces, limits and continuity in metric spaces, Compact sets, Perfect sets, Connected sets.

Review of convergence of sequences and series of numbers. Sequences and Series of functions, Point wise and uniform convergence, term by term differentiation and integration of series of functions.
Power series-convergence and their properties.Equicontinuity, Pointwise and uniform boundedness, Ascoli's theorem, Weierstrass approximation theorem.

Riemann-Stieltje's integral: Definition and existence of the integral, Properties of the integral.

Text Books:

1. W. Rudin, Principles of Mathematical Analysis McGraw Hill Book Co, 1976.
2. R.G. Bartle, The Elements of Real Analysis, 2nd Ed., J .Wiley, NY, London, 1964.

Reference Books:

1. R.R.Goldberg, Methods of Real Analysis, Wiley, 1976.
2. Kenneth A. Ross, Elements of Analysis: The Theory of Calculus, Springer Verlag, UTM, 1980.
3. S. R. Ghorpade and B. V. Limaye, Introduction to Calculus and Real Analysis, Springer, 2006.

MAL513 - Theory of Ordinary Differential Equations **[(3-0-0); Credit: 3]**

Objective: The general purpose of this course is to introduce basis concepts of theory of ordinary differential equations, to give several methods including the series method for solving linear and nonlinear differential equations, to learn about existence and uniqueness of the solution to the nonlinear initial value problems.

First order differential equations, Linear differential equations of higher order, Linear dependenceand Wronskian, Basic theory for linear equations, Method of variation of parameters, Linear equations with variable coefficients.

Systems of differential equations, existence and uniqueness theorems, Fundamental matrix, Non-homogeneous linear systems, Linear systems with constant coefficients and periodic coefficients, Existence and uniqueness of solutions, Gronwall inequality, Successive approximation, Picard's theorem, Nonuniqueness of solutions, Continuous dependence on initial conditions, Existence of solutions in the large.

Boundary Value Problems: Green's function, Sturm-Liouville problem, eigenvalue problems.

Stability of linear and nonlinear systems: Lyapunov stability, Sturm's Comparison theorem

Text Books:

1.S.G.Deo and V. Raghavendra: Ordinary Differential Equations, Tata McGraw Hill Pub. Co., New Delhi, 1997.

2. E.A.Coddington : Introduction to Ordinary Differential Equations, Prentice Hall of India, 1974.

3. D. A. Sanchez, Ordinary Differential Equations and Stability Theory: An Introduction, Dover Publ. Inc.,New York, 1968.

Reference Books:

1. M. Rama Mohana Rao: Ordinary differential equations -Theory and applications. Affiliated East West Press, New Delhi, 1980.

2. G.F. Simmons, Differential equations: Theory, Technique and practice, McGraw Hill.

MAL514 -Discrete Mathematics **[(3-0-0); Credit: 3]**

Objective: The objective of this subject is to expose student to understand the importance Discrete Mathematical Structures in science and engineering.

Sets and propositions: Combinations of sets, Finite and Infinite sets, Uncountably infinite sets, Principle of inclusion and exclusion, Mathematical induction. Propositions, Fundamentals of logic, First order logic, Ordered sets.

Permutations, Combinations, Numeric functions, Generating functions.

Recurrence relations and recursive algorithms:Recurrence relations, Linear recurrence relations with constant coefficients, Solution by the method of generating functions, Sorting algorithm.

Relations and functions: Properties of binary relations, Equivalence relations and partitions, Partial and total ordering relations, Transitive closure and Warshal's algorithm.

Boolean algebra : Chains, Lattices and algebraic systems, Principle of duality, Basic properties of algebraic systems, Distributive and complemented lattices, Boolean lattices and algebras, Uniqueness of finite boolean algebras, Boolean expressions and functions.

Graphs and planar graphs : Basic terminology, Multigraphs and weighted graphs, Paths and circuits, Shortest paths in weighted graphs, Eulerian paths and circuits, Hamiltonian paths and circuits. Colourable graphs, Chromatic numbers, Fivecolour theorem and Four colour problem.

Trees and cut-sets : trees, rooted trees, Path lengths in rooted trees, Spanning trees and BFS & DFS algorithms, Minimum spanning trees and Prims &Kruskal's algorithms.

Text Books:

1. C.L.Liu: Elements of Discrete Mathematics, McGraw Hill, 1985.
2. J.P. Tremblay and RManohar : Discrete Mathematical Structures with applications to Computer Science, McGraw Hill Book Co., New Delhi 1975.

Reference Books:

1. J. L. Mott, A. Kandel and T. P. Baker : Discrete Mathematics for Computer Scientists,Reston Pub. Co, 1983.
2. K.D. Joshi: Foundations in Discrete Mathematics, New Age International, 1989.

MAL 521 - Complex Analysis **[(3-0-0); Credit: 3]**

Objective: The objective of this subject is to expose student to understand the importance of complex variables and its applications to science and engineering.

Algebra of complex numbers; Operations of absolute value and Conjugate; Standard inequalities for absolute value, Extended complex plane, Spherical representation.

The exponential and logarithmic functions, Trigonometric functions of a complex variable. Analytic functions as mappings from C to C. Conformality of a map linear fractional transformations and their properties and elementary conformal mappings. Examples.

Complex Integration: Line integrals, rectifiable curves; Cauchy theorem, Index of a closed curves, Cauchy's integral formulae, Cauchy's inequality, Liouville's theorem, Morera’s theorem, Taylor series expansion.

Singularities: Laurent series expansions, Removable singularities, Poles and essentialsingularities, zeros of analytic functions, Identity theorem, Maximum modulus theorem, Schwartz's lemma, Cauchy's residue theorem; Evaluation of real integrals using Cauchy's residue theorem.

Text Books:

1. R.V. Churchill and Brown: Complex variables and applications, McGraw Hill, 1990.
2. Murray Speigel: Complex Variables, Shaum’s Outline Series, 1964.

Reference books:

1. Ahlfors: Complex Analysis, McGraw-Hill, New York, 1953.
2. Conway: Functions of One Complex Variable, Second Edition, NAROSA Publ., 2002
3. Ponnusamy and Silverman: Complex variables with applications, Birkhauser, 2006.

MAL 522 - Topology **[(3-0-0); Credit: 3]**

Objective: The objective of this subject is to provide the student with the concept and the understanding in topological spaces and compact spaces.

Topological spaces, Basis for a topology, Subspace topology, Closed Sets.
Limit points, Continuous functions, The Product topology, Metric topology, Quotient topology. Connected Spaces, Connected subspaces of R.
Component and path components, Path connectedness, Compact Spaces, Compactness in metric spaces, Local compactness, One point compactification, The countability and Separation axioms, Uryshon's Lemma, Uryshon'smetrization theorem,Tietz extension theorem, The Tychonoff Theorem.

Text books:

1. J. R. Munkres: Topology, 2nd edition, Pearson Education India, 2001.
2. K. D. Joshi: Introduction to general Topology, New Age Internations, New Delhi, 2000.

Reference Books:

1. G. F. Simmons: Introduction to topology and modern analysis, International student edition, 1963.
2. J. V. Deshpande: Introduction to Topology, Tata McGraw Hill, 1988.

MAL 523 - Algebra **[(3-0-0); Credit: 3]**

Objective: The objective of this subject is to expose student to understand several important concepts in abstract algebra, including group, ring, field, homomorphism, isomorphism, and quotient structure, and to apply some of these concepts to real world problems.

Binary operation, and its properties, Definition of a group, Examples and basicproperties.Subgroups, Coset of a subgroup, Lagrange's theorem.
Cyclic groups, Order of a group, Normal subgroups, Quotient group.Homomorphisms, Kernel Image of a homomorphism, Isomorphism theorems.
Permutation groups, Cayley's theorems,Direct product of groups. Group action on a set, Semi-direct product.Sylow' theorems.Structure of finite abelian groups.Applications, Some nontrivialExamples.
Rings: definition, Examples and basic properties. Zero divisors,Integral domains, Fields, Characteristic of a ring, Quotient field of an integraldomain. Subrings, Ideals, Quotient rings, Isomorphism theorems.Ring of polynomlals, Prime, Irreducible elements and their properties, UFD, PID and Euclidean domains. Prime ideal, Maximal ideals, Prime avoidance theorem,Chinese remainder theorem.

Text book:

1. S. Lang: Algebra Third edition, Addison Wesley,1999.
2. D.S. Dummitand R. M. Foote: Abstract Algebra, 2nd Ed., John Wiley, 2002.

References Books:

1. M. Artin, Algebra: Prentice Hall of India, 1994.
2. J.A. Gallian: Contemporary Abstract Algebra, 4th Ed., Narosa, 1999.
3. N. Jacobson: Basic Algebra I, 2nd Ed., Hindustan Publishing Co., 1984, W.H. Freeman, 1985.

MAL 524 - Partial Differential Equations **[(3-0-0); Credit: 3]**

Objective: The objective of this subject is to expose student to understand the importance of partial differential equations.

Formulation, Linear and quasi-linear first order partial differential equations, Paffian equation, Condition for integrability, Lagrange's method for linear equations. First order non-linear equations, method of Charpit - method of characteristics.

Equations of higher order : Method of solution for the case of constant coefficients, Equations of second order reduction to canonical forms, Characteristic curves and the Cauchy problem, Riemann's method for the solution of linear hyperbolic equations, Monge's method for the solution of non-linear second order equations,Method of solution by separation of variables.

Laplace's equations: Elementary solutions, Boundary value problems, Green's functions for Laplace's equation, Solution using orthogonal functions.

Wave equations: One dimensional equation and its solution in trigonometric series, Riemann-Volterra solution, vibrating membrane.

Diffusion equations: Elementary solution, Solution in terms of orthogonal functions.

Text Books:

1. I.N. Sneddon: Elements of Partial Differential Equations, McGraw Hill, New York, 1957.
2. Ioannis P Stavroulakis, Stepan A Tersian : Partial Differential Equations: An Introduction with MATHEMATICA and MAPLE, World scientific, Singapore, 2004.

Reference Books:

1. P. Prasad, Renuka Ravindran: Partial Differential Equations, New Age International, 1985.
2. T. Amaranath: An elementary course in Partial differential eqations, Narosa Pub., New Delhi, 2009.

MAL525 - Numerical Analysis **[(3-0-0); Credit: 3]**

Objective: The objective of this subject is to expose student to understand the importance of Numerical methods and the analysis behind it.

Interpolation : Existence, Uniqueness of interpolating polynomial, Error of interpolation, Unequally spaced data; Lagrange's, Newton's divided difference formulae. Equally spaced data: finite difference operators and their properties, Gauss's forward and backward formulae, Inverse interpolation,Hermite interpolation.

Differentiation: Finite difference approximations for first and second order derivatives. Integration: Newton-cotes closed type methods; Particular cases, Error terms, Newton cotes open type methods, Romberg integration, Gaussian quadrature; Legendre, Chebyshev formulae.

Solution of nonlinear and transcendental equations, Regula-Falsi, Newton-Rapson method, Chebyshev's, method, Muller's method, Birge-Vita method, Solution of system of nonlinear equations.

Approximation :Norms, Least square (using monomials and orthogonal polynomials), Uniform and Chebyshev approximations.

Solution of linear algebraic system of equations: LU Decomposition, Gauss-Seidal methods; solution of tridiagonal system. Ill conditioned equations.

Eigen values and Eigen vectors: Power and Jacobi methods.

Solution of Ordinary differential equations: Initial value problems: Single step methods; Taylor's, Euler's, Runge-Kutta methods, Error analysis; Multi-step methods: Adam-Bashforth, Nystorm's, Adams- Moulton methods, Milne's predictor-corrector methods. System of IVP's and higher order IVP's.

Text Books:

1. M.K. Jain, S.R.K. Iyengar and R.K. Jain : Numerical Methods for Engineers and Scientists, New Age International, 2003.
2. C.F. Gerald, P.O. Wheatley: Applied Numerical Analysis, Addison-Wesley, 1994.

Reference Books:

1. D. Aitkinson : Numerical Analysis, John Wiley and Sons, 2009.
2. Samuel D. Cante and Carl de Boor: Elementary Numerical Analysis, McGraw Hill, 1980.

MAL526 - Numerical Computation Laboratory **[(0-0-2); Credit: 1]**

Objective: The objective of this subject is to expose student to practise the numerical methods on computer.

Programs Based on Numerical Analysis using FORTRAN/C/C++.

Simple programs dealing with fundamental concepts in Fortran programs using conditional statements, Do loops, Subscripted variables, Function subprograms and subroutines.

Programs for solution of quadratic equation, Solution of algebraic and transcendental equations, Gauss-Seidel method, Inverse of a matrix/Gaussian elimination etc.,

Numerical integration , Euler's and modified Euler's methods, Runge-Kutta methods, Tridiagonal system by Thomas algorithm.

MAL 531 - Functional Analysis **[(3-0-0); Credit: 3]**

Objective: This course is intended to introduce the student to the basic concepts and theorems of functional analysis and its applications.

Normed linear spaces, Banach spaces, Hilbert Spaces and basic properties, Heine Borel theorem, Riesz lemma and best approximation property.

Inner product spaces, Projection Theorem; Orthonormal bases, Bassel inequality and parseval's Formula, Riesz Fischer theorem.

Bounded operators, Space of bounded operators and dual space, Riesz representation theorem, Adjoint of operators on a Hilbert Space; unbounded operators, Convergence of sequence of operators.

Hahn Banach extension theorem, Uniform boundedness principle, Closed graph theorem and open mapping theorem, Applications.

Text books:

1. M. T. Nair, Functional Analysis, A first course, Prentice Hall of India, 2002.
2. B. V. Limaye, Functional Analysis, Second Edition, New Age International, 1996.

Reference Books:

1. E. Kreyzig,Introduation to Functional Analysis with Applications, Wiley, 1989.
2. Bollobas, Linear Analysis, Cambridge university Press (Indian Edition)1999.
3. A .E. Taylor and D. C. Lay, Introduction to Functional analysis, 2nd edition, Wiley, New York, 1980.
4. G. F. Simmons: Introduction to Topology, Introduction to Topology and Modern Analysis, International student edition, 1963.

MAL 532 - Operations Research **[(3-0-0); Credit: 3]**

Objective: The objective of this subject is to expose student to understand the importance optimization techniques and the theory behind it.

Linear Programming : Lines and hyperplanes, Convex sets, Convex hull, Formulation of a Linear Programming Problem, Theorems dealing with vertices of feasible regions and optimality, Graphical solution.

Simplex method (including Big M method and two phase method), Dual problem, Duality theory, Dual simplex method, Revised simplex method.

Transportation problem, Existence of solution, Degeneracy, MODI method (including the theory). Assignment problem, Travelling salesman problem.

Nonlinear programming problem: Constrained NLPP, Lagrange's multipliers method - Convex NLPP, Kuhn-Tucker conditioins (including the proof), Quadratic programming (Wolfe's and Beale's methods)

Queuing theory: Characteristics of queueing systems, Birth and death process, Steady state solutions, Single server model (finite and infinite capacities), Single server model (with SIRO), Models with state dependent arrival and service rates, Waiting time distributions.

Text Books:

1. J.C. Pant: Introduction to Optimization: Operations Research, Jain Brothers, 1988.
2. H.A.Taha: Operations Research, An Introduction , PHI, 1987.

Reference Books:

1. H.M.Wagner: Principles of Operations Research, Prentice Hall of India, Delhi, 1982.
2. Wayne L. Winston: Operations research: Applications and algorithms, Cengage learning Indian edition, 2003.
3. S.S. Rao: Engineering Optimization: Theory & Practice , New Age International(P) Limited, New Delhi, 1996.

MAL 533 - Fluid Dynamics **[(3-0-0); Credit: 3]**

Objective: The objective of this subject is to expose student to understand the importance of fluid dynamics in Science & Engineering.

Kinematicsof fluids in motion: Real fluids and ideal fluids, Velocity of a fluid at a point , Stream lines and path lines, Steady and unsteady flows, The velocity potential, The velocity vector, Local and particle rates of change, Equation of continuity, Acceleration of fluid , Conditions at a rigid boundary. Equations of motion of fluid: Euler's equations of motion, Bernoulli's equation, Some flows involving axial symmetry, Some special two-dimensional flows.

Some three dimensional flows: Introduction, Sources, sinks and doublets, Axisymmetric flows, Stokes' stream function. The Milne-Thomson circle theorem, Theorem of Blasius.

Viscous flows: Stress analysis in fluid motion, Relations between stress and rate of strain, Coefficient of viscocity and laminar flow, Navier-Stokes' equations of motion of viscous fluid, Steady motion between parallel planes and through tube of uniform cross section.Flow between concentric rotating cylinders.

Steady viscous flow in tube having uniform elliptic cross section, Tube having equilateral triangular cross section, Steady flow past a fixed sphere.

Text Books:

1. F. Chorlton, Text book of Dynamics, CBS Publishers and Distributors, Delhi, 1998.
2. Whitaker, Introduction to Fluid Mechanics, Prentice-Hall, 1968.

Reference books:

1. N. Curle and H. J. Davies : Modern Fluid Dynamics,Vol. I, 1968.
2. P.K. Kundu and I.M. Cohen, Fluid Mechanics (3rd edition) Elsevier Science & Technology, 2002.
3. L.M.Milne Thomson : Theoretical Hydrodynamics, Macmillan Company, New York, 1955.
4. G.K. Batchelor, Introduction to Fluid Dynamics, Cambridge University Press, 1967.

MAL 534 - Probability& Statistics **[(3-0-0); Credit: 3]**

Objective: The objective of this subject is to expose student to understand the importance of probability and statistical methods.

Axiomatic definition, properties conditional probability, Independent events, Baye's theorem, Density function, Distribution function, Expectation, Moments, Moment generating function, Characteristic function, Chebyshev's inequality, Law of large numbers.

Special distributions: Binomial, Negative Binomial, Geometric, Poisson, Uniform, Normal, Gamma, Exponential, Joint distributions, Marginal and conditional density functions.

Sampling theory for small and large samples, Sampling distributions, Estimation theory and interval estimation for population parameters using normal, t, F and Chi square distributions. Testing of hypothesis and test of significance.

Text Books:

1. V.K. Rohatgi and A.K.M. Ehsanes Sateh : An Introduction to Probability and Statistics, John Wiley & Sons, 2004.
2. J.J. Miller and Freund: Probability and Statistics for Engineering, Person education, 2005.

Reference books:

1. K.S. Trivedi, Probability Statistics with Reliability, Queuing and Computer Science applications, Prentice Hall of India, 1982.
2. M.R. Spiegel, Theory and problems of Probability and statistics, McGraw-Hill Book, 2000.

MAD 501 - Project Phase-I **[Credit: 1]**

Objective: The objective of the project is to expose student to understand the importance research in mathematics.

Topic will be decided by the guide and student.

MAL 541 - Measure Theory and Integration **[(3-0-0); Credit: 3]**

Objective: The objective of this subject is to expose student to understand the importance of measure theory and Integration.

Review of Riemann-Stieltje's integral: Algebras of sets - Borel subsets of R -Lebesgue outer measure and its properties, Algebras of measurable sets in R - nonmeasurable set, Example of measurable set which is not a Borel set - Lebesgue measure and its properties, Measurable functions, Point wise convergence and convergence in measure,Egoroff theorem.

Lebesgue integral, Lebesgue criterion of Riemann integrability, Fatou's lemma, Convergence theorem, Differentiation of an integral, Absolute continuity with respect to Lebesgue measure. Lebesgue integral in the plane, Fubini's theorem.

Text Books:

1. H.L.Royden, Real Analysis, Macmillan, 1968.
2. de Barra, Measure Theory and Integration, Wiley Eastern Ltd., 1981.

Reference Books:

1. I.K. Rana: An Introduction to Measure and Integration, Second Edition,Narosa, 2005.
2. D.L. Cohn: Measure Theory , Birkhauser, 1997.
3. P.K. Jain and V.P. Gupta: Lebesgue Measure and Integration, New Age International, 2006.

MAL 542 - Integral Transforms & Integral Equations **[(3-0-0); Credit: 3]**

Objective: The objective of this subject is to expose student to understand the importance of transform techniques like Laplace transforms, Fourier Transforms, Z transforms and Integral equations.

Integral Transforms: Laplace transforms: Definitions, Properties, Laplace transforms of some elementary functions, Convolution Theorem, Inverse Laplace transformation, Applications.

Fourier transforms:Definitions,Properties, Fourier transforms of some elementary functions, Convolution theorems, Fourier transform as a limit of Fourier Series, Applications to PDE.

Integral Equations: Volterra Integral Equations: Basic concepts, Relationship between linear differential equations and Volterra integral equations,Resolvent Kernel of Volterra Integral equation, Solution of integral equations by Resolvent Kernel, Method of successive approximations, Convolution type equations, Solution of integral differential equations with the aid of Laplace transformation.

Fredholm Integral equations: Fredholm equations of the second kind, Fundamentals - Iterated Kernels, Constructing the resolvent Kernel with the aid of iterated Kernels, Integral equations with degenerate kernels, Characteristic numbers and eigenfunctions, Solution of homogeneous integral equations with degenerate kernel, Nonhomogeneous symmetric equations,Fredholm alternative.

Text Books:

1. I.Sneddon, The Use of Integral Transforms (Tata McGraw Hill), 1974.
2. Hildebrand, Methods of Applied Mathematics, Dover Publications; 2nd edition, 1992.

Reference Books:

1. Krasnov, Problems and Exercises in Integral Equations (Mir Publ.), 1971.
2. Ram P Kanwal, Linear Integral Equations (Academic Press), 1971.
3. F.G.Tricomi, Integral Equations, Dover Publications, (1985).
4. J.L. Schiff, The Laplace Transform, Springer (1999)
5. M.R. Spiegel, Laplace Transforms (Schaum's Series), McGraw-Hill, 1965.

MAD 502 - Project Phase –II **[(4-0-0); Credit: 4]**

Objective: The objective of the project is to expose student to understand the importance research in mathematics.

MAL 535 - Relativity (Elective) **[(3-0-0); Credit: 3]**

Objective: The objective of this subject is to expose student to understand the importance of Relativity.

Introduction of Tensors and Tensor calculus, Covariant differentiation, Parallel transport, Riemann curvature tensor, Geodesics.
Principles of special relativity, Line interval, Proper time, Lorentz transformation, Minkowskispacetime, Lightcones, Relativistic momentum 4-vectors, Lorentz transformation of electromagnetic field.
Principle of equivalence, Connection between gravity and geometry, Metric tensor and its properties, Concept of curved spaces and spacetimes, Tangent space and four vectors, Particle trajectories in gravitational field.Einstein's field equations, Definition of the stress tensor, Bianchi identities and conservation of the stress tensor, Einstein's equations for weak gravitational fields, The Newtonian limit.
Derivation of Schwarz'schild metric, Basic properties of Schwarzschild metric coordinate-systems, Effective potential for particle orbits in Schwarzschild metric, general properties Precession of perihelion, Deflection of ultra relativistic particles, Gravitational red-shift.
Models of the universe, Friedmann-Robertson-Walker models, Hubble's law, Angular size. Source counts, Cosmological constant, Horizons. The early universe, Thermodynamics of the early universe.
Microwave background.Observational constraints, Measurement of Hubble's constant, Anisotropy of large-scale Age of the universe, Dark matter.

Text Books:

1. J. V. Narlikar : An Introduction to Relativity, Cambridge University Press (Indian edition by Macmillan company of India Ltd.) 2010.
2. Ray d'Inverno : Introducing Einstein's Relativity, Clarendon Press, 1996

Reference Books:

1. J.V. Narlikar: An Introduction to Cosmology, Cambridge University Press, 2002
2. James B. Hartle: Gravity : An Introduction to Einstein's General Relativity, Pearson Education, 2003

MAL 536 - Numerical Solution of Differential Equations (Elective) [(3-0-0); Credit: 3]

Objective: The objective of this subject is to expose student to understand the importance of finite difference methods for solving ordinary and partial differential equations.

Ordinary Differential Equations: Multistep (Explicit and Implicit) Methods for Initial Value problems, Stability and convergence analysis, Linear and nonlinear boundary value problems, Quasilinearization. Shooting methods.

Finite Difference Methods : Finite difference approximations for derivatives, boundary value problems with explicit boundary conditions, Implicit boundary conditions, Error analysis, Stability analysis, Convergence analysis.

Cubic splines and their application for solving two point boundary value problems.

Partial Differential Equations: Finite difference approximations for partial derivatives and finite difference schemes for Parabolic equations : Schmidt's two level, Multilevel explicit methods, Crank-Nicolson's two level, Multilevel implicit methods, Dirichlet's problem, Neumann problem, Mixed boundary value problem. Hyperbolic Equations : Explicit methods, implicit methods, One space dimension, two space dimensions, ADI methods. Elliptic equations : Laplace equation, Poisson equation, iterative schemes, Dirichlet's problem, Neumann problem, mixed boundary value problem, ADI methods.

Text Books:

1. M.K.Jain: Numerical Solution of Differential Equations, Wiley Eastern, Delhi, 1983.
2. G.D.Smith: Numerical Solution of Partial Differential Equations, Oxford University Press, 1985.

Reference Books:

1. P.Henrici : Discrete variable methods in Ordinary Differential Equations, John Wiley, 1964.
2. A.R. Mitchell: Computational Methods in Partial Differential equations, John Wiley & Sons, New York, 1996.
3. Steven C Chapra, Raymond P Canale: Numerical Methods for Engineers, Tata McGraw Hill, New Delhi, 2007.

MAL537 - Mathematical Modeling (Elective) **[(3-0-0); Credit: 3]**

Objective: The objective of this subject is to expose student to understand the importance of Mathematical modelling.

Need Techniques, Classification and Simple Illustrations. Mathematical modeling through ordinary differential equations and partial differential equations.

Mathematical Modeling Through Graphs. Mathematical modeling through functional integral delay, Differential and differential-difference equations, Mathematical modeling through calculus of variations and dynamic programming, Mathematical modeling through mathematical programming, maximum principle and minimum entropy principle.

Multivariable optimization models, Computational methods for optimization models, Introduction to probability models, Stochastic models.

Text Books:

1. J.N. Kapur : Mathematical Modeling , New Age International Pub, 1988.
2. Mark M. Meerschaert: Mathematical Modeling, Elsevier Publ., 2007.

Reference Books:

1. Edward A. Bender: An introduction to mathematical Modeling, CRC Press,2002
2. Walter J. Meyer, Concepts of Mathematical Modeling, Dover Publ., 2000.

MAL 543 - Operator theory (Elective) **[(3-0-0); Credit: 3]**

Objective: The objective of this subject is to expose student to understand the importance of Operator theory.

Dual space considerations: Representation of duals of the spaces c_{00} with p-norms, c_0 and c with supremum-norm, l_p C[a,b] and L_p. Reflexivity, Weak and weak* convergences. Best Approximation in Reflexive spaces.

Operators on Banach and Hilbert spaces: Compact operators and its properties; Integral operators as compact operators; Adjoint of operators between Hilbert spaces; Self-adjoint, Normal and unitary operators; Numerical range and numerical radius; Hilbert--Schmidt operators.

Spectral results for Banach and Hilbert space operators: Eigen spectrum, Approximate eigen spectrum, Spectrum and resolvent; Spectral radius formula; Spectral mapping theorem; Riesz-Schauder theory, Spectral results for normal, Self-adjoint and unitary operators; Functions of self-adjoint operators.

Spectral representation of operators: Spectral theorem and singular value representation for compact self-adjoint operators; Spectral theorem for self-adjoint operators.

Text book:

1. M. T. Nair: Functional Analysis: A first course, Prentice Hall of India, 2002.
2. B. V. Limaye: Functional Analysis, Second Edition, New Age Internationals, 1996.

References:

1. E. Kreyzig: Introduation to Functional Analysis with Applications, Wiley, 1989.
2. Bollobas: Linear Analysis, Cambridge university Press, 1999.

MAL544 - Finite Element Method (Elective) [(3-0-0); Credit: 3]

Objective: The objective of this subject is to expose student to understand the importance of finite element method.

Calculus of Variations: Variational methods.
Introduction and motivation, Weak formulation of BVP and Galerkin approximation, Piecewise polynomial spaces and finite element method. Results from Sobolev spaces, Variational formulation of elliptic BVP, Lax-Milgram theorem, Estimation for general FE approximation, Construction of FE spaces, Polynomial approximation theory in Sobolev spaces, Variational problem for second order elliptic operators and approximations, Mixed methods, Iterative techniques.

Computer implementation of FEM: Basic of finite element approximation; Mesh generation; Global problem issue; systems of linear equations; Sparse systems.

Text Books:

1. Barna Szabo and Ivo Babuska: Finite Element Analysis, John Wiley And Sons, Inc..
2. D. Braess, Finite elements, Cambridge university press, 1997.
3. J.N.Reddy : An introduction to the Finite Element Method, McGraw Hill.

Reference Books:

1. S. C. Brenner and L. R. Scott, The Mathematical Theory of Finite Element Methods, Springer.

MAL545 - Numerical Methods for Hyperbolic Problems (Elective) [(3-0-0); Credit: 3]

Objective: The objective of this subject is to expose student to understand the importance of Computational fluid dynamics.

Linear hyperbolic equations: Finite differences, Theoretical concepts of stability and consistency, Order of accuracy, Upwind, Lax Fredrichs and Lax-Wendroff schemes.

Nonlinear equations:One dimensional scalar conservation laws, Review of basic theory, Solutions of the Reimann problem and entropy conditions. First order schemes like Lax Fredrichs, Godunov, Enquist-Osher and Roe's scheme. Convergence results, Entropy consistency and numerical viscosity.

Introduction to the higher order schemes: Lax-Wendroff scheme, Upwind schemes of Van Leer, ENO schemes, Central schemes, Relaxation methods. Introduction to the finite volume methods.

Text Books:

1. E. F. Toro: Reimann Solvers and Numerical Methods for Fluid Dynamics, Springer, 1999.
2. R. J. LeVeque: Numerical Methods for Conservation Laws, BirkhauserVerlag, 1992.

Reference Books:

1. C. B. Laney: Computational Gas Dynamics. Cambridge university press, 1998.
2. Randall J. LeVeque: Finite Volume Methods for Hyperbolic Problems, Cambridge Texts in Applied Mathematics, 2002.

MAL546 - Bio Mechanics (Elective) **[(3-0-0); Credit: 3]**

Objective: The objective of this subject is to expose student to understand the importance of Bio mechanics.

Fundamental concepts of Biomechanics. Cardiovascular system: Basic concepts about blood, blood vessels, governing equations, models on blood flow, flow in large blood vessels, microcirculation, pulsatile flow, stenotic region flow.

Peristalsis: Basic concepts, governing equations, peristaltic transport under long wave length approximation, peristaltic flow for small amplitudes and small Reynold's number.

Flow in Renal tubules: Basic concepts, governing equations, ultra filteration, flow through proximal tubules, flow through tubes with varying cross section.

Text Book:

1. Y.C.Fung, Biomechanics, Springer-Verla, Springer; 2nd edition , 1996.
2. J.N.Kapur, Mathematical Models in Biology and Medicine, Affiliated East West Press, New Delhi, 1985.

Reference Books:

1. C.G.Caro, T.J.Pedley, R.Schroter : Mechanics of circulation, Oxford University Press., 1978.
2. Susan J Hall,.Basic Biomechanics. Boston: McGraw-Hill Companies, Inc., 1999.

MAL547 - Multivariate Data Analysis (Elective) **[(3-0-0); Credit: 3]**

Objective: The objective of this subject is to expose student to understand the importance of multivariate analysis.

Introduction: The Organization of Data, Distance Matrix Algebra and Random vectors, Mean vectors and Covariance matrices, Matrix Inequalities and Maximization.

Sample Geometry and Random Sampling, Geometry of the sample, Expected values of the sample mean and Covariance matrix, Generalized variance, Sample mean, Covariance and correlation as Matrix operations. Multivariate Normal distribution, Sampling from a multivariate Normal Distribution, Maximum Likelihood Estimation, The sampling distribution of $\overline{\mathbf{X}}$ and S.

Inferences about a Mean Vector, Plausibility of $\mu_{\mathbf{0}}$ as a value for a Normal population Mean, Hotelling's $\mathbf{T}^{\mathbf{2}}$ and Likelihood Ratio Tests Paired Comparisons and a Repeated Measures Design, Comparing Mean Vectors from two populations Introduction to One Way MANOVA.

Multivariate linear Regression Models, Least Squares Estimation, Inferences about the Regression Model, Multivariate Multiple Regression.

Principal Components, Population Principal Components, Graphing the Principal Components.
The Orthogonal Factor Model, Factor rotation, Factor scores, Perspectives and a strategy for factor analysis.

Text Books:

1. Richard A. Johnson, Applied Multivariate Statistical analysis, Prentice Hall, 2001.
2. Dean W. Wichern, PHI, 3rd Edition – 1996

Reference Books:

1. Cooly WW and PR Lohnes – Multivariate Data Analysis-John Wiley & Sons, 1971.
2. J E Hair, W C Black, B. J. Babin, R E Anderson, Multivariate Data Analysis, Prentice Hall, 2009.

MAL448 - Financial Mathematics **[(3-0-0); Credit: 3]**

Objective:

The main objective of this course is to introduce the financial mathematics and is applications to marketing.

Some Basic definitions and terminology.

Basic Theory of Option Pricing: Single and Multi-Period Binomial Pricing Models, Cox-Ross-Rubinstein (CRR) Model, Black-Scholes Formula for Option Pricing as a Limit of CRR Model.

Brownian and Geometric Brownian Motion, Theory of Martingales. Stochastic Calculus, Stochastic Differential Equations, Ito's Formula to Solve SDE's. Applications of Stochastic Calculus in Option Pricing.

Mean-Variance Portfolio Theory: Markowitz Model of Portfolio Optimization and Capital Asset Pricing Model (CAPM). Limitations of Markowitz Model and New Measures of Risk.

Interest Rates and Interest Rate Derivatives: Binomial Lattice Model, Vasicek, Hull and White Models for Bond Pricing.

Text books:

1. D. G. Luenberger: Investment Science, Oxford University Press.
2. M. Capińsky and T. Zastawniak: Mathematics for Finance: An Introduction to Financial Engineering, Springer.
3. Thomas Mikosch: Elementary Stochastic Calculus with Finance in view, World Scientific.
4. Suresh Chandra, S. Dharmaraja, Aparna Mehra, R. Khemchandani: Financial Mathematics: An Introduction, Narosa Publishing House.

Reference Books:

1. S. E. Shreve: Stochastic Calculus for Finance, Vol. I & Vol. II, Springer.
2. Sean Dineen: Probability Theory in Finance: A Mathematical Guide to the Black-Scholes Formula, American Mathematical Society, Indian edition.

MAL 549 - Nonlinear Dynamical Systems (Elective) [3-0-0; Credits 3]

Continuous dynamical systems, Equilibrium points. Linearization. Hartman-Grobman theorem. Linear stability analysis. Liapunov Theorem and global stability. Stable, unstable and central manifolds: Stable and central manifold theorems. Phase-plane analysis. Constructing phase plane diagrams. Hamiltonian systems, conservative systems, dissipative systems. Limit cycles: Existence and uniqueness of limit cycle. Poincare-Bendixson theorem.

Bifurcations: Saddle-node, transcritical, pitchfork(supercritical and subcritical), Hopf-Andronov (Supercritical and subcritical), omoclinic, Heteroclinic and Bogdanov-Takens bifurcations (Normal forms and their applications).

Strange attractors and Chaos: Lorentz system, Lienard systems, Henon map, Rossler system and food chain model. Constructing phase bifurcation diagrams. Cascades of periodic doubling bifurcation. Lyapunov exponents.

Books:

1. L. Perko,Differential equations and dynamical systems, volume 7. Springer, 2000.
2. Steven H. Strogatz,Nonlinear Dynamics and Chaos with applications to Physics, Biology, Chemistry and Engineering, Levant Books, 2007.
3. S. Wiggins,Introduction to Applied Nonlinear Dynamical Systems and Chaos, Springer, Volume 2, 2003.
4. M.W. Hirsch, S. Smale, and R.L. Devaney,Differential equations, dynamical systems and an introduction to chaos, volume 60. Academic Press, 2004.
5. Yuri A. Kuznetsov,Elements of applied bifurcation Theory, Springer, 2004

MAL 550 – Numerical Linear Algebra (Elective) **[(3-0-0); Credit: 3]**

Objective: The objective of this subject is to expose student to understand the basic importance of Linear Algebra and numerical linear algebra and its applications to science and engineering.

Special Matrices, Vector and Matrix Norms, SVD. Floating Point Numbers and Errors. Stability, Conditioning and Accuracy. Gauss Elimination and Linear Systems, LU Factorization using Gaussian Elimination, Stability of Gaussian Elimination, Basic Results on Existence and Uniqueness, Some Applications Giving Rise to Linear Systems of Problems, LU Factorization Methods, Conditioning and Pivoting, Inverses and Determinants. Iterative Methods for Large and Sparse Problems: Gauss Seidal, SOR, Chebyshev Acceleration, Conjugate Gradient Method, Preconditioning. QR Factorization, SVD, and Least Squares Solutions. Numerical Eigenvalue Problems, Generalized Eigenvalue Problem.

Text Books:

1. G. H. Golub and C. F. van Loan: Matrix Computations, Johns Hopkins University Press, 1984.
2. L. N. Trefethen and D. Bau, III: Numerical Linear Algebra, SIAM, 1997.
3. G. Allaire and S. M. Kaber: Numerical Linear Algebra, Springer, 2007.
4. B. N. Datta: Numerical Linear Algebra and Applications, Springer, 2008.

Contact Courses for Ph.D. Scholars of Mathematics Department

MAL 601 - Singular Perturbation Theory [3-0-0; Credits 3]

Mathematical preliminaries:
Little-*oh*, Big-*oh*, Asymptotically equal to or behaves like notations, Asymptotic sequences and asymptotic expansions, Convergent series versus divergent series, Asymptotic expansions with a parameter, Uniformity or breakdown, Intermediate variables and the overlap region, the matching principle, matching with logarithmic terms, composite expansions.

Introductory applications:
Roots of equations, integration of functions represented by asymptotic expansions, ordinary differential equations: regular problems, simple singular problems, scaling of differential equations, equations which exhibit a boundary layer behavior, where is the boundary layer? boundary layers and transition layers.

The method of multiple scales:
Nearly linear oscillations, nonlinear oscillators, applications to classical ordinary differential equations, WKB method for slowly varying oscillations, turning point problem, applications to partial differential equations, limitation on the use of the method of multiple scales, boundary layer problems.

Some physical applications of singular perturbation problems:

Books:

1. **R.S. Johnson** , Singular perturbation theory : Mathematical and analytical techniques with applications to engineering , Springer, 2005.
2. **Ali Hasan Nayfeh**, Perturbation methods, John-Wiley & Sons, New-York, 1972.
3. **Carl M. Bender, Steven A. Orszag**, Advanced mathematical methods for scientists and engineers: Asymptotic methods and perturbation theory, Springer, 2010.
4. **O'Malley, R.E.** : Introduction to Singular Perturbations, Academic Press, New York, 1974.
5. **O'Malley, R.E.** : Singular Perturbation Methods for Ordinary Differential Equations, Springer-Verlag, New York, 1991.
6. **Smith, D.R. :** Singular Perturbation Theory – An Introduction with applications, Cambridge University Press, Cambridge, 1985.
7. **Van Dyke, M. :** Perturbation Methods in Fluid Mechanics, Academic Press, New York, 1964.
8. **Kevorkian, J. and Cole, J.D. :** Perturbation Methods in Applied Mathematics, Springer-Verlag, New York, 1981.

MAL 602 - Fitted Numerical Methods for Singular Perturbation Problems [3-0-0; Credits 3]

Motivation for the study of singular perturbation problems, simple examples of singular perturbation problems.

Uniform numerical methods for problems with initial and boundary layers:

Initial value problems- some uniformly convergent difference schemes, constant fitting factors, optimal error estimates. Boundary value problems- constant fitting factors for a self adjoint problem, non self adjoint problem, self adjoint problem in conservation form, non self adjoint problem in conservation form, problems with mixed boundary conditions, fitted versus standard method, experimental determination of order or uniform convergence.

Simple fitted mesh methods in one dimension, convergence of fitted mesh finite difference methods for linear convection-diffusion problems in one dimension, linear convection-diffusion problems in two dimensions and their numerical solutions, fitted numerical methods for problems with initial and parabolic boundary layers.

Books

1. **Doolan, E.P., Miller, J.J.H. and Schilders, W.H.A. :** Uniform Numerical Methods for problems with Initial and Boundary Layers, Boole Press, Dublin, 1980.
2. **Miller, J.J.H., O'Riordan, E. and Shishkin, G.I. :** Fitted Numerical methods for singular perturbation problems, World Scientific, River Edge, NJ, 1996.

MAL 603 - Advanced Fluid Dynamics **[(3-0-0); Credit: 3]**

Equations governing the motion of viscous fluid:
Viscosity, Newton law of viscosity; stress and rate of strain tensors; Law of conservation of mass, continuity equations; law of conservation of momentum, Navier-Stokes equations for viscous fluids; law of conservation of angular momentum; law of conservation of energy and energy equation.

Boundary conditions:
No slip boundary condition, slip boundary condition.

Some simple viscous fluid flows:
Steady motion between parallel planes and through tube of uniform cross section. Flow between concentric rotating cylinders. Flow of two immiscible fluids (with different viscosities) between parallel plates, flow of two immiscible fluids with different viscosities through a circular tube.

Steady viscous flow in tube having uniform elliptic cross section, Tube having equilateral triangular cross section, Steady flow past a fixed sphere.

Unsteady flows:
Unsteady unidirectional flows above flat plate due to impulsive motion of flat plate; unsteady unidirectional flows between two parallel plates due to impulsive motion of plates; unsteady unidirectional flow through circular cylindrical pipe due the impulsive motion of the pipe, unsteady unidirectional flow through concentric cylinders due to the impulsive motion of the cylinders; unsteady unidirectional flows of two immiscible fluids between parallel plates due to impulsive motion of the plates; unsteady unidirectional flows of two immiscible fluids through cylindrical pipe due to the impulsive motion of the pipe.

Flows with low and high Reynolds number:
Reynolds number (Re), flows with negligible inertia (Re<<1), flows with high Reynolds number (Re>>1).

Books:

1. F. Chorlton, Text book of Dynamics, CBS Publishers and Distributors, Delhi, 1998.
2. I.G.Currie, Fundamental Mechanics of Fluids, Marcel Dekker, Inc.: New York. 2003.
3. W. E. Langlois, M.O.Deville, Slow Viscous Flow (2nd Edition), Springer, 2014.
4. S.Whitaker, Introduction to Fluid Mechanics, Prentice-Hall, 1968.
5. G.K. Batchelor, Introduction to Fluid Dynamics, Cambridge University Press, 1967.
6. D.J.Acheson, Elementary Fluid Dynamics, Oxford University Press
7. H. Schlichting, K. Gersten, Boundary Layer Theory, Springer Science & Business Media, 2000.
8. Y.A.Cengel, J.M.Cimbala, Fluid Mechanics: Fundamentals and Applications, Tata McGraw-Hill Publishing Company Ltd., New Delhi, 2006

MAL604 - Matrix Iterative Analysis **[(3-0-0); Credit: 3]**

1. Matrix Properties and Concepts:

Norms and Spectral Radii, Bounds for the Spectral Radius of a Matrix and Directed Graphs, Diagonally Dominant Matrices, Ovals of Cassini.

2. Nonnegative Matrices:

Spectral Radii of Nonnegative Matrices, Cyclic and Primitive Matrices, Reducible Matrices, Nonnegative Matrices and Directed Graphs.

3. Basic Iterative Methods and Comparison Theorems:

The Point Jacobi, Gauss-Seidel, and Successive Overrelaxation Iterative Methods, Average Rates of Convergence, The Stein-Rosenberg Theorem, The Ostrowski-Reich Theorem, Stieltjes Matrices, M-Matrices and H-Matrices, Regular and Weak Regular Splittings of Matrices.

4. Successive Overrelaxation Iterative Methods:

P-Cyclic Matrices, The Successive Over relaxation, Iterative Method for p-Cyclic Matrices, Theoretical Determination of an Optimum Relaxation Factor.

Text Books:

1. Matrix Iterative Analysis, Richard S. Varga, Second Edition, Springer Berlin Heidelberg.

MAL 605 - Theory of Micropolar and Couplestress fluids [(3-0-0); Credit: 3]

(Pre-requisites: Fluid Dynamics)

Kinematics of Fluids Flow: Introduction, Velocity Gradient Tensor, Rate of Deformation Tensor, Analysis of Strain Rates, Spin Tensor, Curvature-Twist Rate Tensor, Objective Tensors, Balance of Mass.

Governing Equations: Introduction, Measure of Mechanical Interactions, Euler's Laws of Motion, Stress and Couple Stress Vectors, Stress and Couple Stress Tensors, Cauchy's Laws of Motion, Analysis of Stress, Energy Balance Equations, Entropy Inequality

Couple Stress Fluids: Introduction, Constitutive Equations, Equations of Motion, Boundary Conditions, Steady Flow between Parallel Plates, Steady Flow between Two Co-axial Cylinders, Poiseuille Flow through Circular Pipe, steady immiscible flows between parallel plates, steady immiscible flows through cicular cylindrical pipe.

Micro Fluids: Introduction, Description of Micromotion, Kinematics of Deformation, Conservation of Mass, Balance of Momenta, Microinertia Moments, Balance of Energy, Entropy Inequality, Constitutive Equations for Micro Fluids, Linear Theory of Micro Fluids, Equations of Motions.

Micropolar Fluids: Introduction, Skew-symmetric of the Gyration Tensor and Microisotropy, Micropolar Fluids, Thermodynamics of Micropolar Fluids, Equations of Motion, Boundary and Initial Conditions, Two Limiting Cases, Steady Flow between Parallel Plates, Steady Couette Flow between Two Co-axial Cylinders, Pipe Poiseuille Flow, steady immiscible flows between parallel plates, steady immiscible flows through circular cylindrical pipe.

Reference Books/Articles:

1) Vijay Kumar Stokes, Theory of Fluids with Microstructure – An Introduction, 1984, SpringerVerlag.
2) R. Aris, Vectors, Tensors, and the Basic Equations of Fluid Mechanics, 1990, Dover Publications Inc.
3) A.C. Eringen, Microcontinuum Field Theories I Foundations and Solids, 1999, Springer.
4) A.C. Eringen, Microcontinuum Field Theories II Fluent Media, 2001, Springer.
5) G. Lukaszewicz, Micropolar Fluids Theory and Applications, 1999, Birkhauser Boston.
6) R.K. Rathy, An Introduction to Fluid Dynamics, 1976, Oxford & IBH Publishing.
7) William F. Hughes, John A. Brighton, Fluid Dynamics, 3rd Ed., 2004, Tata McGraw- Hill.
8) M. Devakar, N. Ch. Ramgopal, Fully Developed Flows of Two Immiscible Couple stress and Newtonian Fluids Through non-Porous and Porous Medium in a Horizontal Cylinder, Journal of Porous Media, 18 (2015), 549-558.
9) M. Devakar, N. Ch. Ramgopal, Unidirectional Flows of Two Immiscible Micropolar and Newtonian Fluids Through non-Porous and Porous Medium in a Horizontal Circular Cylinder, Preprint, 2015.
10) J. C. UMAVATHI, J. PRATHAP KUMAR, Ali J. CHAMKHA, Convective Flow of Two Iimmiscible Viscous and Couplestress Permeable Fluids Through a Vertical Channel, Turkish J. Eng. Env. Sci., 33 (2009) , 221 – 243.
11) J. Prathap Kumar, J.C. Umavathi, Ali J. Chamkha, Ioan Pop, Fully Developed Free Convective Flow of Micropolar and Viscous Fluids in a Vertical Channel, Applied Mathematical Modelling, 34 (2010), 1175–1186.
12) J.V. Ramana Murthy, J. Srinivas, Second Law Analysis For Poiseuille Flow of Immiscible Micropolar Fluids in a Channel, International Journal of Heat and Mass Transfer , 65 (2013), 254–264.

MAL606 - Cosmology (Elective) **[(3-0-0) ; Credits : 3]**

Note: The pre-requisite for this course is MAL 535 – Relativity

Objective: The objective of this subject is to expose student to understand the importance of cosmology which deals origin, structure formation and evolution of the universe.

An overview of the large scale structure of the universe. Einstein's modified field equations with the cosmological term. Static cosmological models of the Einstein and de-Sitler; their derivation, geometrical and physical properties and comparison with the actual universe. Hubble's law, non-static cosmological models, cosmological principles and Weyl's postulate. Derivation of the Robertson-Walker metric and its geometrical properties. Hubble and deceleration parameters. Red shift in the Robertson-Walker geometry. Einstein's equations for the Robertson-Walker metric, fundamental dynamical equations of the standard big-bang cosmology-Friedman Robertson-Walker models. Initial singularity-the big bang, density and pressure in the present universe. Critical density- the open, closed and flat universes. Age of the universe. The radiation and matter dominated era of the universe. The red shift versus distance relation. Event and particle horizons. Observational constraints, Measurement of Hubble's constant, Dark matter and Dark energy.

Recommended Books:

1. R. C. Tolman, Relativity, Thermodynamics and Cosmology, Clarendon Press, Oxford, 1934.

2. S. Weinberg, Gravitation and Cosmology, John Wiley, 1972.

3. J. V. Narlikar, Introduction to Cosmology, Cambridge University Press, 1998.

4. J. N. Islam, An Introduction to Mathematical Cosmology, Cambridge University Press, 1999.

5. J. A. Peacock, Cosmological Physics, Cambridge University Press, 1999

MAL607 - Fixed Point Theory and Applications **[(3-0-0); Credit: 3]**

(Pre-requisites: Real Analysis, Functional Analysis, Topology)

Fundamentals: Topological spaces, Normed spaces, Dense set and separable space, Linear operators, Space of bounded linear operators, Hahn-Banach theorem and applications Compactness, Reflexivity, Weak topologies, Continuity of mappings, Convexity,

Smoothness, Duality Mappings: Strict convexity, Uniform convexity, Convex functions Smoothness, Banach limit, Metric projection and retraction mappings,

Geometric Coefficients of Banach Spaces: Asymptotic centers and asymptotic radius, The Opial and uniform Opial conditions, Normal structure

Existence Theorems in Metric Spaces: Contraction mappings and their generalizations, Multivalued mappings, Convexity structure and fixed points, Normal

structure coefficient and fixed points, Lifschitz's coefficient and fixed points

Existence Theorems in Banach Spaces: Non-self contraction mappings, Nonexpansive mappings, Multivaluednonexpansive mappings, Asymptotically nonexpansive mappings,

Uniformly L-Lipschitzian mappings, Non-Lipschitzian mappings, Pseudocontractive mappings

Approximation of Fixed Points: Basic properties, Convergence of successive iterates, Mann iteration process, Nonexpansive and quasi-nonexpansive mappings

The modified Mann iteration process, The Ishikawa iteration process, The S-iteration process

Strong Convergence Theorems: Convergence of approximants of self-mappings, Convergence of approximants of non-self mappings, Convergence of Halpern iteration process

Applications of Fixed Point Theorems: Attractors of the IFS, Best approximation theory, Solutions of operator equations, Differential and integral equations, Variational inequality, Variational inclusion problem

Books:

1. Ravi P. Agarwal, DonalO'Regan, and D.R. Sahu, Fixed Point Theory for Lipschitzian-type Mappings with Applications, Springer New York 2009.

2. Mohamed A. Khamsi and William A. Kirk, An Introduction to Metric Spaces and Fixed Point Theory, John Wiley &Sons, inc.2001.