Visvesvaraya National Institute of Technology, Nagpur
Department of Mathematics
Assignment-5 (Maxima, Minima and Lagrange multiplier method) **Subject: MAL-102**

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1. Examine the following functions for local maxima, local minima and saddle points
(i) $4xy - x^4 - y^4$ (ii) $x^3 - 3xy$

2. Let $f(x, y) = 3x^4 - 4x^2y + y^2$. Show that f has a local minimum at $(0, 0)$ along every line through $(0, 0)$. Does f have a minimum at $(0, 0)$? Is $(0, 0)$ is a saddle point for f ?

3. Find the shortest distance from the point $(1, 0, -2)$ to the plane $x + 2y + z = 4$.

4. Assume that among all rectangular boxes with fixed surface area of 20 square meters, there is a box of largest possible volume. Find its dimension.

5. A cardboard box without lid is to have a volume $32,000cm^3$. Find the dimensions that minimize the amount of the cardboard used.

6. Find a point on the curve $y^2 = (x - 1)^3$ which is nearest to the origin in $\mathbb{R}^2$.

7. Divide the number A into three parts such that the product of first number, square of second number and cube of third number is the maximum.

8. Find the points on the sphere $x^2 + y^2 + z^2 = 4$ that are nearest to and farthest from the point $(3, 1, -1)$.

9. L&T produces steel boxes at 3 different plants in amounts x, y and z respectively, producing an annual revenue of $R(x, y, z) = 8xyz^2 - 200(x + y + z)$. The company is to produce 100 units annually. How should production be distributed to maximize revenue?

10. Show that the greatest rectangular parallelepiped that can be inscribed in an ellipsoid
$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1 \text{ is } \frac{8abc}{3\sqrt{3}}.$$

11. Maximize the quantity $f(x, y, z) = \dfrac{5xyz}{x + 2y + 4z}$ subject to constraint $xyz = 8$.

12. If x, y, z are lengths of the perpendiculars dropped from a point inside the triangle of given area A, on the three sides of the triangle then the minimum value of $x^2 + y^2 + z^2$ is $\dfrac{4A^2}{a^2 + b^2 + c^2}$.

13. Find the point nearest to the origin in $\mathbb{R}^3$ and lying on the line which is the intersection of the planes $x + 2y + 3z = 6$ and $x + 3y + 9z = 9$.

14. The stationary values of $u(x, y, z) = \dfrac{x^2}{a^4} + \dfrac{y^2}{b^4} + \dfrac{z^2}{c^4}$ subject to the constraints $lx + my + nz = 0$ and $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} + \dfrac{z^2}{c^2} = 1$ are the roots of the equation $\dfrac{l^2a^4}{1 - a^2u} + \dfrac{m^2b^4}{1 - b^2u} + \dfrac{n^2c^4}{1 - c^2u} = 0$.