Visvesvaraya National Institute of Technology, Nagpur
Department of Mathematics
Assignment-4 (Functions of several variables) **Subject: MAL-102**

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1. If the sides (a, b and c) and angles (A, B and C) of a plane triangle vary in such a way that the circum radius remains constant, then prove that $\dfrac{da}{\cos A} + \dfrac{db}{\cos B} + \dfrac{dc}{\cos C} = 0$.

2. Calculate approximate value of $\ln[(1.03)^{\frac{1}{3}} + (0.98)^{\frac{1}{4}} - 1]$.

3. If $x^2 + y^2 + z^2 - 2xyz = 1$ then show that $\dfrac{dx}{\sqrt{1-x^2}} + \dfrac{dy}{\sqrt{1-y^2}} + \dfrac{dz}{\sqrt{1-z^2}} = 0$.

4. State and prove Taylor's theorem for real-valued functions of two variables.

5. Use Taylor's theorem at the origin, to find quadratic and cubic approximations of the following functions: (i) xe^y (ii) $y \sin x$ and (iii) $e^x \ln(1+y)$.

6. Use Taylor's theorem to find quadratic approximation of (i) $f(x,y) = \cos x \cos y$ and (ii) $f(x,y) = e^x \sin y$ at the origin. Also esitimate the error in the approximation if $|x| \le 0.1$ and $|y| \le 0.1$.

7. If $x = a\cos\xi\cosh\eta$; $y = a\sinh\xi\sin\eta$ then show that $\frac{\partial(x,y)}{\partial(\xi,\eta)} = \frac{a^2}{2}[\cosh 2\xi - \cos 2\eta]$.

8. If $F = xu + v - y$; $G = u^2 + vy + w$; $H = zu - v + vw$ then find $\frac{\partial(F,G,H)}{\partial(u,v,w)}$?

9. If $x = r\sin\theta\cos\phi$, $y = r\sin\theta\sin\phi$, and $z = r\cos\theta$ then find $\frac{\partial(x,y,z)}{\partial(r,\theta,\phi)}$?

10. If u, v are functions of two independent variables x and y then prove that $\frac{\partial(u,v)}{\partial(x,y)} \times \frac{\partial(x,y)}{\partial(u,v)} = 1$.

11. If u, v are implicit functions of two independent variables x and y connected by the relations $f_1(u,v,x,y) = 0$; $f_2(u,v,x,y) = 0$ then prove that $\frac{\partial u}{\partial x} = -\dfrac{\frac{\partial(f_1,f_2)}{\partial(x,v)}}{\frac{\partial(f_1,f_2)}{\partial(u,v)}}$, $\frac{\partial u}{\partial y} = -\dfrac{\frac{\partial(f_1,f_2)}{\partial(y,v)}}{\frac{\partial(f_1,f_2)}{\partial(u,v)}}$, $\frac{\partial v}{\partial x} = -\dfrac{\frac{\partial(f_1,f_2)}{\partial(u,x)}}{\frac{\partial(f_1,f_2)}{\partial(u,v)}}$ and $\frac{\partial v}{\partial y} = -\dfrac{\frac{\partial(f_1,f_2)}{\partial(u,y)}}{\frac{\partial(f_1,f_2)}{\partial(u,v)}}$.

12. If $u^3 + v + w = x + y^2 + z^2$, $u + v^3 + w = x^2 + y + z^2$ and $u + v + w^3 = x^2 + y^2 + z$. Then prove that $\frac{\partial(u,v,w)}{\partial(x,y,z)} = \frac{1-4(xy+yz+zx)+16xyz}{2-3(u^2+v^2+w^2)+27u^2v^2w^2}$.

13. If $u^3 + v^3 + w^3 = x + y + z$, $u^2 + v^2 + w^2 = x^3 + y^3 + z^3$ and $u + v + w = x^2 + y^2 + z^2$, then find $\frac{\partial(u,v,w)}{\partial(x,y,z)}$?

14. Check whether the following are functionally dependent? If so, find the functional relation: i) $u = \frac{x-y}{x+z}$; $v = \frac{x+z}{y+z}$ ii) $u = x + y + z$, $v = x^2 + y^2 + z^2$, $w = xy + yz + zx$.

15. If $f_k = \frac{x_k}{1+x_1+x_2+x_3}$ for $k = 1, 2, 3$ then prove that $\frac{\partial(f_1,f_2,f_3)}{\partial(x_1,x_2,x_3)} = (1 + x_1 + x_2 + x_3)^{-4}$.