Visvesvaraya National Institute of Technology, Nagpur
Department of Mathematics
Assignment-3 (Functions of several variables) **Subject: MAL-102**

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1. If $u = (x^2 + y^2 + z^2)^{m/2}$ find $m \neq 0$ such that $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0$.

2. Find $x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2}$, where $u(x,y) = x^2 \phi(\frac{y}{x}) + y\psi(\frac{y}{x})$ and ϕ, ψ are arbitrary functions.

3. Let $u = F(x-y, y-z, z-x)$, then show that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$.

4. If $f(x,y) = \phi(u,v)$ and $u = x^2 - y^2, v = 2xy$ then prove that $\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = 4(x^2 + y^2)\left(\frac{\partial^2 \phi}{\partial u^2} + \frac{\partial^2 \phi}{\partial v^2}\right)$.

5. If $u = f\left(\frac{x}{y}, \frac{y}{z}, \frac{z}{x}\right)$ then find $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z}$.

6. If $u = \tan^{-1}\left(\dfrac{x^3 + y^3}{x - y}\right)$ then show that
   (i) $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \sin(2u)$ and (ii) $x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = \sin(4u) - \sin(2u)$

7. If $u = \csc^{-1}\left(\dfrac{x^{\frac{1}{2}} + y^{\frac{1}{2}}}{x^{\frac{1}{3}} + y^{\frac{1}{3}}}\right)^{\frac{1}{2}}$ then show that $x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = \frac{\tan u}{12}\left(\frac{13}{12} + \frac{\tan^2 u}{12}\right)$.

8. If $u = f(x,y)$ where $x = r\cos\theta, y = r\sin\theta$. Show that $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = \frac{\partial^2 u}{\partial r^2} + \frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2} + \frac{1}{r}\frac{\partial u}{\partial r}$.

9. If $z = e^x \sin y$, where $x = st^2$ and $y = s^2 t$. Find $\frac{\partial z}{\partial s}$ and $\frac{\partial z}{\partial t}$.

10. If $z = f(x,y)$ where f is differentiable, $x = g(t)$ and $y = h(t)$. Given $g(3) = 2, g'(3) = 5, h(3) = 7, h'(3) = -4, f_x(2,7) = 6$ and $f_y(2,7) = -8$, then find $\frac{dz}{dt}$ when $t = 3$.

11. If $u = f(x,y)$ where $x = e^s \cos t$ and $y = e^s \sin t$, show that
    (i) $\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2 = e^{-2s}\left[\left(\frac{\partial u}{\partial s}\right)^2 + \left(\frac{\partial u}{\partial t}\right)^2\right]$, (ii) $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = e^{-2s}\left[\frac{\partial^2 u}{\partial s^2} + \frac{\partial^2 u}{\partial t^2}\right]$

12. Find the equation of the tangent plane to the surfaces at the specified points
    (i) $z = 9x^2 + y^2 + 6x - 3y + 5, \quad (1,2,18)$ (ii) $z = y\cos(x-y), \quad (2,2,2)$.

13. Find the equation of the tangent plane and normal line, (i) at the point $(-2,1,-3)$ to the ellipsoid $\frac{x^2}{4} + y^2 + \frac{z^2}{9} = 3$ and (ii) at the point $(1,0,0)$ to surface $z + 1 = xe^y\cos(z)$.

14. Find the total differential of the functions
    (i) $z = x^2 + 3xy - y^2$ (ii) $v = y\cos(xy)$ (iii) $u = e^{-t}\sin(s + 2t)$