Visvesvaraya National Institute of Technology, Nagpur
Department of Mathematics

Assignment-2: Partial Derivatives **Subject: Mathematics-II(MAL102)**

1. Prove that the function $f(x,y) = \begin{cases} \frac{x^2+y^2}{x-y} & \text{for } (x,y) \neq (0,0) \\ 0 & \text{for } (x,y) = (0,0) \end{cases}$ is not continuous at $(0,0)$. Do you think the first order partial derivatives exists for this function at the point $(0,0)$? Justify your answer.

2. Do you think the function$f(x,y) = \begin{cases} 1 & \text{for } xy \neq 0 \\ 0 & \text{for } xy = 0 \end{cases}$ is continuous at the origin? What can you say about the existence of first order partial derivatives of this function at $(0,0)$? Justify your answer.

3. Prove or disprove that the functions $i) f(x,y) = \sqrt{x^2+y^2}$, $ii)\ f(x,y) = |xy|$ and $iii)\ f(x,y) = \sqrt{|xy|}$ are differentiable at the origin?

4. Find $f_x$and f_y for the functions: $i)\ z = f(x)g(y)$ and $ii)\ f(x,y) = \int_y^x \cos(t^2)dt$.

5. The temperature at a point (x,y) on a flat plate is given by $T(x,y) = \frac{60}{1+x^2+y^2}$ where T is measured in centigrade and x, y in meters. Find the rate of change of temperature with respect to distance at the point $(2,1)$ in $a)$ the $x-$ direction and $b)$ the $y-$ direction.

6. If $f(x,y) = x\left(x^2+y^2\right)^{-\frac{3}{2}} e^{\sin\left(x^2 y\right)}$ then find $f_x(1,0)$.

7. Is there a function $f(x,y)$ whose partial derivatives are $f_x(x,y) = x+4y$ and $f_y(x,y) = 3x-y$? Explain?

8. If $z\,(x+y) = x^2+y^2$ then show that $\left(\frac{\partial z}{\partial x} - \frac{\partial z}{\partial y}\right)^2 = 4\left[1 - \frac{\partial z}{\partial x} - \frac{\partial z}{\partial y}\right]$.

9. If $u = \log\left(x^3+y^3+z^3-3xyz\right)$ then prove that
$i)\ \frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = \frac{3}{(x+y+z)}$ and $ii)\ \left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}\right)^2 u = -\frac{9}{(x+y+z)^2}$.

10. If $\theta = t^n e^{-\left(\frac{r^2}{4t}\right)}$then find the value of n for which $\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial \theta}{\partial r}\right) = \frac{\partial \theta}{\partial t}$?

11. Find the values of a and b such that the function $u(x,t) = e^{ax}\sin(x+bt)$ may be the solution of the equation $\frac{\partial^2 u}{\partial x^2} = \frac{1}{k}\frac{\partial u}{\partial t}$ satisfying the condition that $u = 0$ as $x \to \infty$.

12. Prove the following:
$i)$ If $u = \left(1-2xy+y^2\right)^{-\frac{1}{2}}$ then $\frac{\partial}{\partial x}\left((1-x^2)\frac{\partial u}{\partial x}\right) + \frac{\partial}{\partial y}\left(y^2\frac{\partial u}{\partial y}\right) = 0$.
$ii)$ If $u = f(r)$, where $r = \sqrt{x^2+y^2}$ then $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = f''(r) + \frac{1}{r}f'(r)$.
And also find $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}$ if $r = \sqrt{x^2+y^2+z^2}$.
$iii)$ If $\frac{x^2}{a^2+u} + \frac{y^2}{b^2+u} + \frac{z^2}{c^2+u} = 1$ then $\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2 + \left(\frac{\partial u}{\partial z}\right)^2 = 2\left(x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z}\right)$.

13. Check whether the following functions can be the solutions of Laplace equation $(i.e., u_{xx} + u_{yy} = 0)$:
$i)\ u(x,y) = e^x \sin y$ and $ii)\ u(x,y) = \sin x \cosh y + \cos x \sinh y$.

14. If f and g are twice differentiable functions of single variable.
Show that the function $u(x,t) = f(x+at)+g(x-at)$ is a solution of the 2-dimensional wave equation $u_{tt} = a^2 u_{xx}$ and hence show that $\frac{t}{(a^2t^2-x^2)}$ is a solution of wave equation.

15. If $x^x y^y z^z = c$ then prove that $\frac{\partial^2 z}{\partial x \partial y} = -(x\log ex)^{-1}$ at $x = y = z$.

16. If $u = \log(\tan x + \tan y + \tan z)$ then show that $\sin 2x\frac{\partial u}{\partial x} + \sin 2y\frac{\partial u}{\partial y} + \sin 2z\frac{\partial u}{\partial z} = 2$.

17. If $u = f(x,y)$ then show that the Laplace equation remains invariant under the rotation of coordinate axes.
(Hint: $x = \xi\cos\alpha - \eta\sin\alpha,\ y = \xi\sin\alpha + \eta\cos\alpha$)