Visvesvaraya National Institute of Technology, Nagpur
Department of Mathematics
Assignment-1: Multiple Integrals

Double integral **Subject: MAL-102**

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1. Evaluate $\int\int_D xydA$ where D is the region bounded by the line $y = x-1$ and the parabola $y^2 = 2x+6$.
2. Find the volume of the tetrahedran bounded by the planes $x + 2y + z = 2, x = 2y, x = 0$ and $z = 0$.
3. Compute the volume of the solid enclosed by the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$.
4. Show that (i) $\int_0^1 \left(\int_0^1 \frac{x^2-y^2}{x^2+y^2} dy \right) dx = 0$ (ii) $\int_0^1 \left(\int_0^1 \frac{x^2-y^2}{x^2+y^2} dx \right) dy = 0$.
5. Prove that (i) $\int_0^1 \left(\int_0^1 \frac{x-y}{(x+y)^3} dy \right) dx = \frac{1}{2}$ (ii) $\int_0^1 \left(\int_0^1 \frac{x-y}{(x+y)^3} dx \right) dy = -\frac{1}{2}$. Does the double integral $\int\int_R \frac{x-y}{(x+y)^3} dA$ exist over $R = [0,1] \times [0,1]$?
6. Prove the Dirichlet's formula: $\int\int_S x^{p-1} y^{q-1} dxdy = \frac{\Gamma(\frac{p}{2})\Gamma(\frac{q}{2})}{4\Gamma(\frac{p}{2} + \frac{q}{2} + 1)}$, whether (i) $p \geq 1, q \geq 1$, or (ii) $0 < p < 1, 0 < q < 1$, where S is the region bounded by the first quadrant of the circle $x^2 + y^2 = 1$.
7. Evaluate $\int\int_R f(x,y)dxdy$ over the rectangle $R = [0,1] \times [0,1]$, where $f(x,y) = \begin{cases} x+y & \text{if } x^2 < y < 2x^2 \\ 0 & \text{elsewhere} \end{cases}$.
8. Evaluate $\int\int_R [x+y] dxdy$, over the rectangle $R = [0,1] \times [0,2]$, where $[x+y]$ denotes the greatest integer less than or equal to $(x+y)$.
9. Prove that $\int\int_R \sqrt{|y - x^2|} dxdy = \frac{3\pi + 8}{6}$, where $R = [-1,1] \times [0,2]$.
10. Evaluate $\int\int_D \log(x^2 + y^2) dxdy$, where D is the region in the first quadrant lying between the circles $x^2 + y^2 = 1$ and $x^2 + y^2 = 4$.
11. (i) Evaluate $\int\int_{D_a} e^{-(x^2+y^2)} dxdy$, where D_a is the disk $x^2 + y^2 \leq a^2$.
    Hence deduce that $\int\int_{\mathbb{R}^2} e^{-(x^2+y^2)} dxdy = \pi$, further show that $\int_{\mathbb{R}} e^{-x^2} dx = \sqrt{\pi}$ (which is the gaussian integral).
    (ii) Find $\int_{\mathbb{R}} e^{-2x^2} dx$.
12. Prove that $\int\int_D x^{m-1} y^{n-1} (1-x-y)^{p-1} dxdy = \frac{\Gamma(m)\Gamma(n)\Gamma(p)}{\Gamma(m+n+p)}$, where D is region bounded by the line $x + y = 1$ and the coordinate axis.
13. Consider $D = \{(x,y) \in \mathbb{R}^2 : x \geq 0, y \geq 0 \text{ and } 1 \leq 2(x+y) \leq 2\}$ and $f : D \to \mathbb{R}$ defined by $f(x,y) = \frac{y}{x+y}$ for $(x,y) \in D$. Find $\int\int_D f(x,y)dA$ (by change of variable).