VISVESVARAYA NATIONAL INSTITUTE OF TECHNOLOGY
DEPARTMENT OF MATHEMATICS
END SEMESTER RE-EXAMINATION, DECEMBER-2014

Subject: NA & PT (MAL 205) Marks: 60, Duration: 3 hours.
i) Answer any 5 questions from each section.
ii) Each question of Section A carry 2 marks.

Section A

1. (a) Find the condition for convergence of general iteration method.

(b) Find the real root of the equation $xe^x - 3 = 0$ by Regula Falsi method, correct to two decimal places.

(c) Find the smallest eigen value in magnitude of the matrix
$A = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$ using four iterations of the inverse power method.

(d) Suppose that $X(t)$ is a process with $\mu(t) = 3$, $R(t_1, t_2) = 9 + 4e^{-0.2|t_1 - t_2|}$.
Find the mean, variance and covariance of random variables $Z = X(5)$ and $W = X(8)$, where $t_1 = 5$ and $t_2 = 8$.

(e) Calculate mean and variance of the Binomial Distribution.

(f) Let x be a random variable with probability mass function given by
$$f(x) = \frac{1}{x(x+1)}; \quad x = 1, 2, \dots$$
$$0; \quad \text{otherwise}$$
Find the moment generating function of random variable x.

Section B

2. (a) Using Shooting method, solve the boundary value problem
$$u'' = u + 1 \qquad 0 < x < 1$$
$$u(0) = 0, \qquad u(1) = e - 1.$$
Use Taylor's method of order two with $h = 0.25$ to solve the resulting system of first order initial value problems. [6]

(b) Solve the system of equations
$$x_1 + x_2 + x_3 = 1; 4x_1 + 3x_2 - x_3 = 6; 3x_1 + 5x_2 + 3x_3 = 4$$
by Crout's method. [4]

3. (a) Solve the boundary value problem
$$y'' = xy$$
$$y(0) + y'(0) = 1, \quad y(1) = 1.$$
Using Finite difference method with $h = \frac{1}{3}$. [5]

(b) Find $y(0.1), y(0.2), y(0.3)$ from
$$\frac{dy}{dx} = x^2 - y, \quad y(0) = 1.$$
by using Taylor's series method of order two and hence obtain y(0.4) using Adams-Bashforth predictor corrector method: [5]

Predictor Formula: $y_{n+1}^{p} = y_n + \frac{h}{24}[55f_n - 59f_{n-1} + 37f_{n-2} - 9f_{n-3}]$

Corrector Formula: $y_{n+1}^{c} = y_n + \frac{h}{24}[9f_{n+1} - 19f_n - 5f_{n-1} + f_{n-2}].$

4. (a) Consider a random process $X(t)$ defined by $X(t) = A\ cos(\omega t + \theta)$, $-\infty < t < \infty$ where A and ω are constants and θ is a uniform random variable over $(-\pi, \pi)$. Show that $X(t)$ is WSS. [5]

(b) An urn contains three red and two green balls. A random sample of two balls is drawn with replacement. Let $X = 0$ if the first ball drawn is green, $X = 1$ if the first ball drawn is red and let $Y = 0$ if the second ball drawn is green, $Y = 1$ if the second ball drawn is red. Find correlation coefficient. [5]

5. (a) The joint probability density function of (X, Y) is given by
$$f(x, y) = 24xy, \quad x > 0, y > 0, x + y \leq 1,$$
$$0, \quad elsewhere,$$
Find the conditional mean and variance of Y given X. [5]

(b) Suppose that the life in hours of a certain part of radio tube is a continuous random variable X with probability density function given by:
$$f(x) = 100/x^2, \quad \text{when x} \geq 100$$
$$0 \quad , \quad \text{otherwise}$$

(i)What is the probability that all of three such tubes in a given radio set will have to be replaced during the first 150 hours of operation?

(ii)What is the probability that none of three of the original tubes will have to be replaced during that first 150 hours of operation?

(iii)What is the probability that a tube will last less than 200 hours if it is known that the tube is still functioning after 150 hours of service?

(iv)What is the maximum number of tubes that may be inserted into a set so that there is a probability of 0.5 that after 150 hours of service all of them are still functioning? [5]

6. (a) There are 400 students in the first year class of an engineering college. The probability that any student requires a copy of a particular Mathematics book from the college library on any day is 0.1. How many copies of the book should be kept in the library so that the probability may be greater than 0.95 that none of the students requiring a copy from the library has to come back disappointed? (Use normal approximation to the binomial distribution). [5]

(b) Out of 800 families with 4 children each, how many families would be expected to have **(i)** 2 boys and 2 girls**, (ii)** atleast 1 boy,**(iii)** at most 2 girls and **(iv)** children of both sexes. Assume equal probabilities for boys and girls. [5]

7. (a) Determine the discrete probability distribution, expectation, variance, standard deviation of a discrete random variable X which denotes the minimum of the two numbers that appear when a pair of fair dice is thrown once. [5]

(b)The average number of phone calls per minute coming into a switch board between 2 and 4 pm is 2.5. Determine the probability that during one particular minute there will be **(i)** 0; **ii) 1** ; **iii)** 2 **; iv)** 3 and **v)** atmost 5. [5]