

**Visvesvaraya National Institute of Technology, Nagpur**  
**Department of Mathematics**  
**Second Sessional Examination, Winter Semester (2014)**  
**Subject: MAL-205 (Numerical Methods & Probability Theory)**

**Full Marks: 15**

**Duration: 1 hour**

*Note: (i) Answer any five questions. All questions carry equal marks.*

*(ii) Use of non-programmable calculator and statistical tables are permitted.*

(1) Find the eigen value correct to two decimal places which is nearest to 5 for the matrix

$$\begin{bmatrix} 4 & 1 & 0 \\ 1 & 4 & 1 \\ 0 & 1 & 4 \end{bmatrix}$$

using inverse power method. Obtain the corresponding eigen vector. Take the

initial approximate vector as  $V^{(0)} = [1, 1, 1]^T$ .

(2) Derive Euler's modified scheme for the numerical approximation of differential equation

$\frac{dy}{dx} = f(x, y)$  with the initial condition  $y(x_0) = y_0$ . Using this method with  $h = 0.2$ , obtain a

solution of the equation  $\frac{dy}{dx} = \log_{10}(x + y)$ ,  $y(0) = 1$  for  $x = 0.2$ . Perform only two steps of the method..

(3) Use the classical Runge-Kutta method of order four to find the numerical solution at

$x = 0.8$  for the differential equation  $\frac{dy}{dx} = \sqrt{x + y}$ ,  $y(0.4) = 0.41$ . Assume the step length  $h = 0.2$  and round-off the value at the fourth place of decimals.

(4). A bombing plane carrying three bombs flies directly above a railroad track. If a bomb falls within 40 metres of track, the track will be sufficiently damaged to disrupt the traffic. With a certain bomb site the points of impact of a bomb have the probability density function:

$$f(x) = \begin{cases} \frac{100+x}{10,000}, & \text{when } -100 \leq x < 0 \\ \frac{100-x}{10,000}, & \text{when } 0 \leq x < 100 \\ 0, & \text{elsewhere,} \end{cases}$$

where  $x$  represents the vertical deviation (in metres) from the aiming point, which is the track in this case. Find the distribution function. If all the three bombs are used, what is the probability that the track will be damaged?

(5). In a continuous distribution whose relative frequency density is given by

$$f(x) = Ax(2-x), \quad 0 \leq x \leq 2.$$

- a) Find A.
- b) Find the first four moments about the origin ,
- c) Find mean & variance.

(6). A random variable x has the following probability function:-

Values of x	0	1	2	3	4	5	6	7
P(x)	0	k	k	k	2k	K <sup>2</sup>	2k <sup>2</sup>	3k <sup>2</sup>

- (a) Find k.
- (b) Evaluate  $P(x < 6)$  &  $P(x \geq 6)$ .
- (c) If  $P(x \leq a) > \frac{1}{2}$  ,find the minimum value of a.