Visvesvaraya National Institute of Technology, Nagpur

Department of Mathematics

End-Semester Examinations (December 2014)

Numerical Methods & Probability Theory (MAL205)

Time: 3 hrs **Marks: 60**

Note: Section A is compulsory. Answer any five from section B.

Section-A (Answer any Five)

1 **a)** How should the constant α be chosen to ensure the fastest possible convergence with the iteration formula $x_{n+1} = \dfrac{\alpha x_n + x_n^{-2} + 1}{\alpha + 1}$. [2]

b) Using Newton-Raphson iterative method, find the real root of $x \sin x + \cos x = 0$ which is near to $x = \pi$, correct to 3 decimal places. [2]

c) Using Jacobi method find all the eigen values of the matrix $\begin{bmatrix} 1 & \sqrt{2} & 2 \\ \sqrt{2} & 3 & \sqrt{2} \\ 2 & \sqrt{2} & 1 \end{bmatrix}$. [2]

d) Let the variable X have distribution $P(X=0)=P(X=2)=p, P(X=1)=1-2p.$ For what values of p the variance of X is maximum? [2]

e) Find the moment generating function of random variable $X \sim N(\mu, \sigma)$. [2]

f) Define Random sequence & Random process. [2]

Section-B

2. **a)** Solve the following boundary value problem using **shooting** method.

$$u'' = 4(u-1), \quad 0 < x < 1$$

$$u(0) = 2, \qquad u(1) = e^2 + 1$$

with step size $h = 1/3$. Use **Taylor's** method of second order to solve the corresponding system of equations. [7]

b) Solve the system of equations $\begin{bmatrix} 4 & -1 & 0 \\ -1 & 4 & -1 \\ 0 & -1 & 4 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ using Cholesky method. [3]

3. **a)** Solve the following boundary value problem using **finite difference** method.

$$y'' = xy + 1, \quad 0 < x < 1$$

$$y(0) + y'(0) = 1, \qquad y(1) = 1$$

with step size $h = 0.25$. Use **Gauss** elimination method to solve the corresponding system of equations. [5]

b) Using Milne's predictor-corrector method, find $y(0.4)$ for the initial value problem $y' = x^2 + y^2, \ y(0) = 1$ with $h = 0.1$.

Calculate the required intermediate values by **Euler's**method. Perform two iterations of the corrector. [5]

Predictor Formula: $y_{i+1}^{(p)} = y_{i-3} + \dfrac{4h}{3}\left[2f_i - f_{i-1} + 2f_{i-2}\right]$ and

Corrector Formula: $y_{i+1}^{(c)} = y_{i-1} + \dfrac{h}{3}\left[f(x_{i+1}, y_{i+1}^{(p)}) + 4f_i + f_{i-1}\right]$

4. **a)** A petrol pump is supplied with petrol once a day. If its daily volume of sales (x) in thousands of litres is distributed by: $f(x) = 5(1-x)^4, \ 0 \le x \le 1$. [5]

i)Find the average sale of the day. ii) What must be the capacity of its tank in order that the probability that its supply will be exhausted in a given day shall be 0.01?

(b) Given $f(x, y) = e^{-(x+y)}$, $0 \le x, y < \infty$. Are X and Y independent? Find i) $P(X > 1)$, ii) $P(X < Y / X < 2Y)$, iii) $P(1 < X + Y < 2)$. [5]

5. **a)** Show that Poission distribution is a limiting case of Binomial distribution when number of trials is very large and probability of successes is very small. [3]

b) Prove that Mean and Variance of Poission distribution are identical. [2]

c) Two ideal dice are thrown. Let X_1 be the score on the first die and X_2 be the score on the second die. Let $Y = \max(X_1, X_2)$. i) Write down the joint distribution of Y & X_1, and
ii) Find the mean and variance of Y and Covariance(Y, X_1) . [5]

6. **a)** An athlete finds that in the high jump he can clear height of 1.68 m once in five attempts and a height of 1.52 m nine times out of ten attempts. Assuming the heights he can clear in various jumps from a Normal distribution, estimate the mean and standard deviation of the distribution. [4]

b) If the sum of the mean and variance of a binomial distribution of 5 trials is 9/5, find the binomial distribution. Find the probability of i) Atleast 2 success and atmost 03 success. [4]

c) If the probability that an applicant for a driver's license will pass the road test on any given trial is 0.8, What is the probability that he will finally pass the test i)on the fourth trial and ii) in fewer than 4 trials? [2]

7. **a)** The joint p.d.f. of a two-dimensional r.v. (X, Y) is given by $f(x, y) = xy^2 + \frac{x^2}{8}$, $0 \le x \le 2,\ 0 \le y \le 1$. Compute i) $P(X > 1)$, ii) $P\left(Y < \frac{1}{2}\right)$, iii) $P\left(X > 1 / Y < \frac{1}{2}\right)$, iv) $P(X < Y)$, v) $P(X + Y \le 1)$. [5]

b) Given ar.v. Y with characteristic function $\varphi(\omega) = E\{e^{i\omega Y}\}$ and a random process defined by $X(t) = cos(\lambda t + Y)$. Show that $\{X(t)\}$ is stationary in the wide sense if $\varphi(1) = \varphi(2) = 0$. [5]