VISVESVARAYA NATIONAL INSTITUTE OF TECHNOLOGY, NAGPUR
DEPARTMENT OF MATHEMATICS
End Semester Examination - December 2014

III Semester.B.Tech. (Civil & Mining) **Subject: Numerical Analysis** (MAL 202)
Max. Marks: 60 **Duration: 3 hours.**
Date: 04-12-2014

Section A: Answer any five questions from Section A. $5 \times 2 = 10.$

1. (a) Using iteration method, find a real root of the equation $x^3 + x^2 - 1 = 0$ with the initial approximation $x_0 = 0.75$.

   (b) Prove that $\triangle^n e^{ax} = (e^{ah} - 1)^n e^{ax}$.

   (c) Derive Gauss-Legendre two point formula.

   (d) Describe Cholesky method to solve the system of equations $AX = b$ where A is a real symmetric matrix.

   (e) Derive the formula for Runge-Kutta method of order 2.

   (f) Using modified Euler's method obtain the approximate value of $y(0.2)$, given that $\frac{dy}{dx} = x + \sqrt{y}$,$y(0) = 1$.

Section B: Answer any five questions from Section B:

2. (a) Construct the divide difference table for the following data:

x	0.5	1.5	3	5	6.5	8
$f(x)$	1.625	5.875	31	131	282	521

   Also find the interpolating polynomial approximating the data and hence find the value of $f(7)$. (5)

   (b) Perform one iteration of Newton-Raphson method for the system of equations $x^2 + xy + y^2 = 7, x^3 + y^3 = 9$ with the initial approximation $x_0 = 1.5, y_0 = 0.5$. (5)

3. (a) For the method $f'(x_0) = \frac{-3f(x_0)+4f(x_1)-f(x_2)}{2h} + \frac{h^2}{3}f'''(\xi), x_0 < \xi < x_2,$ determine the optimal value of h using the criteria $\mid R.E. \mid + \mid T.E. \mid = minimum$. Using this method and the value of h obtained from the above criteria, determine an approximate value of $f'(2)$ from the following tabulated values:

x	2	2.01	2.02	2.06	2.12
$f(x)$	0.69315	0.69813	0.70310	0.72271	0.75142

   (5)

   (b) Determine ,a,b,c such that the formula $\int_0^h f(x)dx = h\left\{af(0) + bf(\frac{h}{3}) + cf(h)\right\}$ is exact for polynomial of higher order as possible and determine the order of the truncation error. (5)

4. (a) Solve the system of equations $\begin{bmatrix} 2 & 1 & -4 & 1 \\ -4 & 3 & 5 & -2 \\ 1 & -1 & 1 & -1 \\ 1 & 3 & -3 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 4 \\ -10 \\ 2 \\ -1 \end{bmatrix}$ by LU decomposition method, assuming $u_{ii} = 1, i = 1, 2, 3, 4$. (5)

   (b)Set up the Gauss Sideline iterative scheme for the solution of the system $\begin{bmatrix} 5 & 1 & -2 \\ 3 & 4 & -1 \\ 2 & -3 & 5 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ -2 \\ 10 \end{bmatrix}$ and show that the iteration scheme is convergent and hence find the rat of convergence. (5)

5. (a) Find the smallest eigen value in magnitude of the matrix matrix $A = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$ using four iteration of the inverse power method.
   Take the initial approximate vector as $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$. (5)

   (b) Using the Jacobi method find all the eigen values and the corresponding eigen vectors of the matrix $A = \begin{bmatrix} 1 & \sqrt{2} & 2 \\ \sqrt{2} & 3 & \sqrt{2} \\ 2 & \sqrt{2} & 1 \end{bmatrix}$. (5)

6. (a) Find the three term Taylor series solution for the third order initial value problem $w''' + ww'' = 0, w(0) = 0, w'(0) = 0, w''(0) = 1$. Find the bound on the error for $t \in [0, 0.2]$. (5)

   (b) Solve the initial value problem $y' = -2xy^2, y(0) = 1$ with $h = 0.2$ on the interval $[0, 0.4]$. Use the fourth order classical Runge-Kutta method. (5)

7. (a) Solve the following boundary value problem using second order finite difference method: $y'' = x + y$, $y(0) = y(1) = 0$ with step size $h = 0.25$. (5)

   (b) Solve the boundary value problem $y'' = y, y(0) = 0$ and $y(1) = 1.1752$ by shooting method taking $m_0 = y'(0) = 0.8$ and $m_1 = y'(0) = 0.9$. (5)