VISVESVARAYA NATIONAL INSTITUTE OF TECHNOLOGY, NAGPUR
DEPARTMENT OF MATHEMATICS
II Sessional Examination, October 2014

III Semester, B.Tech(ECE, EEE and MEC) **Max. Marks: 15**
Subject: Integral Transforms & PDE (MAL 201) **Duration: 1 hour**

Note: All questions carry equal marks. Answer any five questions.

1. Solve the initial boundary value problem,
   $u_t = u_{xx},\ 0 < x < \pi, t > 0$; $u(0,t) = 0 = u(\pi,t)$ for $t \geq 0$ and $u(x,0) = 2\sin x - 5\sin 3x, 0 \leq x \leq \pi$.

2. Find the solution of the wave equation with initial displacement $x^2 - 5x$ and initial velocity $e^x \sin^2 x$ with $c = 1$ using d'Alembert's formula. (simplify the solution to maximum extent.)

3. Solve the Laplace equation, $\nabla^2 u(x,y) = 0\ 0 < x < l, 0 < y < k$;
   with $u(0,y) = 0 = u(l,y)$ for $0 \leq y \leq k$ and $u(x,0) = 0, u(x,k) = f(x)$ for $0 \leq x \leq l$.

4. (i) Find the Laplace transform of the function $\int_0^t \frac{\sin u}{u}\, du$.
   (ii) If $L\{\sin\sqrt{t}\} = \frac{\sqrt{\pi}}{2s^{\frac{3}{2}}} e^{-\frac{1}{4s}}$, then find $L\left\{\frac{\cos\sqrt{t}}{\sqrt{t}}\right\}$?

5. (i) Let $f(t)$ be a periodic function on $[0,\infty)$ with fundamental period T.
   Show that $\int_{nT}^{(n+1)T} e^{-st} f(t)dt = e^{-nsT} \int_0^T e^{-st} f(t)dt$ for all $n \in \mathbb{N}$.
   (ii) Find the Laplace transform of the period function $f(t) = \begin{cases} 1, & 0 \leq t \leq 2 \\ -1, & 2 \leq t \leq 4, \end{cases}$ and $f(t+4) = f(t)$ for all $t \geq 0$.

6. Solve the integro-differential equation, $y' - y - 6\int_0^t y(u)du = \sin t,\ y(0) = 2$.

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