Visvesvaraya National Institute of Technology, Nagpur
Department of Mathematics
End Semester Re-Examinations, May 2014

Subject: Mathematics-II (MAL102)

Max. Marks: 60 **Time: 3:00 hrs**

Note: Section A is compulsory. Answer any five Questions from Section A. Answer any five Questions from Section B.
Calculators are not permitted.

Section A: $[5 \times 2 = 10]$

1. (a) If f and g are twice differentiable functions of single variable then show that $u(x,t) = f(x+at) + g(x-at)$ is a solution of the 2-dimensional wave equation $u_{tt} = a^2 u_{xx}$, where a is constant.
   (b) Use the Taylor's theorem at the origin to find quadratic approximation of the function $f(x,y) = e^x \ln(1+y)$.
   (c) Show that the family of curves $y^2 = 4c(x+c)$, where c arbitrary parameter, are self-orthogonal.
   (d) By changing the order of the integration evaluate $\int_0^1 \int_{2y}^2 e^{x^2} dx dy$.
   (e) If $\overrightarrow{V} = v_1(x,y,z)\hat{i} + v_2(x,y,z)\hat{j} + v_3(x,y,z)\hat{k}$ then find div(curl $\overrightarrow{V}$).
   (f) Evaluate $\iint_D e^{x^2+y^2} dx dy$, where D is the unit circle centered at the origin.

Section B

2. (a) Show that the function

$$f(x,y) = \begin{cases} \dfrac{x^3+2y^3}{x^2+y^2}, & (x,y) \neq (0,0) \\ 0, & (x,y) = (0,0) \end{cases}$$

   is continuous at $(0,0)$ but not differentiable at $(0,0)$. [3]
   (b) If F is a function of u, v where $u = x-y$ and $v = xy$ then show that $x\dfrac{\partial^2 F}{\partial x^2} + y\dfrac{\partial^2 F}{\partial y^2} = (x+y)\left[\dfrac{\partial^2 F}{\partial u^2} + xy\dfrac{\partial^2 F}{\partial v^2}\right]$. [4]
   (c) A rectangular box without a lid is to be made from 12 m^2 of cardboard. Find the maximum volume of such a box. [3]

3. (a) Solve the differential equation $x^2 y'' - 5xy' + 13y = 30x^2$. [3]
   (b) Solve the differential equation $y(y^2 - 2x^2)dx + x(2y^2 - x^2)dy = 0$. [3]
   (c) Solve the differential equation $(D^2 + 2)y = x^3 + x^2 + e^{-2x} + \cos 3x$. [4]

4. (a) If $u = \tan^{-1}\left(\dfrac{x^3+y^3}{x-y}\right)$ then show that $x^2 u_{xx} + 2xy u_{xy} + y^2 u_{yy} = \sin 4u - \sin 2u$. [4]
   (b) Find whether the vector field $\overrightarrow{F} = y^2 z^3 \hat{i} + 2xyz^3 \hat{j} + 3xy^2 z^2 \hat{k}$ is conservative. If so, find the scalar potential function ϕ. [4]
   (c) Using divergence theorem find $\iint_S \overrightarrow{F}.\hat{n} dS$, where $\overrightarrow{F} = (2x+3z)\hat{i} - (xz+y)\hat{j} + (y^2+2z)\hat{k}$ and S is the surface of the sphere having centre at $(3,-1,2)$ and radius 3. [2]

5. (a) Find the volume of the solid that lies under the paraboloid $z = x^2 + y^2$, above the xy-plane and inside the cylinder $x^2 + y^2 = 2x$. [5]

(b) Find the centre of gravity of a solid of constant density that is bounded by the parabolic cylinder $x = y^2$ and the planes $x = z,\ z = 0$ and $x = 1$. [5]

6. (a) Evaluate the surface integral $\iint_S \overrightarrow{F}.\hat{n}dS$, where $\overrightarrow{F} = z^2\hat{i} + xy\hat{j} - y^2\hat{k}$ and S is the portion of the cylinder $x^2 + y^2 = 36$ and $0 \leq z \leq 4$ included in the first octant. [5]

   (b) Verify Green's theorem in the plane for $\oint_C (3x^2 - 8y^2)dx + (4y - 6xy)dy$, where C is the boundary of the region bounded by $y = \sqrt{x}$ and $y = x^2$. [5]

7. (a) Verify Stoke's theorem for $\overrightarrow{F} = (x + y)\hat{i} + (2x - z)\hat{j} + (y + z)\hat{k}$ over the surface of a triangular lamina with vertices $(2, 0, 0)$, $(0, 3, 0)$ and $(0, 0, 6)$. [6]

   (b) Find the directional derivative of $V^2 (= \overrightarrow{V}.\overrightarrow{V})$, where $\overrightarrow{V} = xy^2\hat{i} + zy^2\hat{j} + xz^2\hat{k}$ at the point $(2, 0, 3)$ in the direction of the outward normal to the sphere $x^2 + y^2 + z^2 = 14$ at the point $(3, 2, 1)$. [4]

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