

Visvesvaraya National Institute of Technology, Nagpur
Department of Mathematics

Subject: MATHEMATICS-II (MAL 102)

Max. Marks: 15

Slot: B

Time: 1:00 hrs

Attempt any five Questions. Marks are indicated against each Question.

1. Find the particular integral of the following differential equation

$$(x^2 + x)\frac{d^2y}{dx^2} + (2 - x^2)\frac{dy}{dx} - (2 + x)y = x(x + 1)^2,$$

where the complementary function is given by $y_c = c_1e^x + \frac{c_2}{x}$ [3]

2. Find the general solution of differential equation $x^4\frac{d^3y}{dx^3} + 2x^3\frac{d^2y}{dx^2} - x^2\frac{dy}{dx} + xy = 1.$ [3]

3. (a) In what direction from the point (1,4,1), is the directional derivative of $f = x^2yz$ maximum? What is the magnitude of this maximum? [1]

(b) If $\nabla f = 2xyz^3i + x^2z^3j + 3x^2yz^2k$ and $f(1, -2, 2) = 4$ then find f . [2]

4. If $f(x, y) = \tan^{-1}(xy)$, find the approximate value of $f(1.1, 0.8)$ using Taylor's series (i) linear approximations and (ii) quadratic approximations. [3]

5. Find the maximum value of the function $f(x, y, z) = x + 2y + 3z$ on the curve of intersection of the plane $x - y + z = 1$ and the cylinder $x^2 + y^2 = 1.$ [3]

6. Evaluate $\int_0^a \int_0^{\sqrt{a^2-x^2}} \frac{1}{(1+e^y)\sqrt{a^2-x^2-y^2}} dx dy.$ [3]

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