Visvesvaraya National Institute of Technology, Nagpur
Department of Mathematics
II Semester B.Tech. End Semester Examination
Mathematics II (MAL 102)

Max Marks: 60 Date:22 – 04 – 2014
Duration: 3 hour (9.00 AM – 12.00 noon)
Note: ***Section A is Compulsory. Answer any FIVE questions from Section A.***
Answer any FIVE questions from Section B.
Calculators are not permitted.

Section A

1. (a) If $u = \frac{yz}{x}, v = \frac{zx}{y}, w = \frac{xy}{z}$, find $\frac{\partial(u,v,w)}{\partial(x,y,z)}$. (2)
   (b) Expand $x^2y + 3y - 2$ in powers of $(x - 1)$ and $(y + 2)$ using Taylor's theorem. (2)
   (c) Solve $xy(1 + xy^2)\frac{dy}{dx} = 1$. (2)
   (d) If $\bar{r} = x\bar{i} + y\bar{j} + z\bar{k}$ and $r =| \bar{r} |$ then find $div\left(\frac{\bar{r}}{r^3}\right)$. (2)
   (e) Find the volume of the tetrahedron bounded by the planes $x + 2y + z = 2, x = 2y, x = 0$ and $z = 0$. (2)
   (f) Prove that for all $x > 0$, $3\int_0^x \int_0^u u^2 f(t)dtdu = \int_0^x (x^3 - u^3)f(u)du$. (2)

Section B

2. (a) Consider the function $f : \mathbb{R}^2 \to \mathbb{R}$ defined by
$$f(x,y) = \begin{cases} \frac{x^2y^2}{x^2y^2+(x-y)^2} & \text{if } (x,y) \neq (0,0), \\ 0 & \text{if } (x,y) = (0,0). \end{cases}$$
   Show that
   (i) $f(x,y)$ is not continuous at $(0,0)$.
   (ii) the first partial derivatives f_x, f_y exist at $(0,0)$. (3)
   (b) Transform the Laplace equation $\dfrac{\partial^2 u}{\partial x^2} + \dfrac{\partial^2 u}{\partial y^2} = 0$ into polar coordinates. (4)
   (c) Find the minimum value of the function $x + y + z$ subject to the condition $\frac{a}{x} + \frac{b}{y} + \frac{c}{z} = 1$. (3)

3. (a) Find the orthogonal trajectories of the family of circles passing through the points $(0,-2)$ and $(0,2)$. (3)
   (b) The radial displacement u in a rotating disc at a distance r from the axis is given by $r^2\frac{d^2u}{dr^2} + r\frac{du}{dr} - u + kr^3 = 0$, where k is a constant. Solve the equation under the conditions $u = 0$ when $r = 0$, $u = 0$ when $r = a$. (4)
   (c) Using the method of variation of parameters, show that the general solution of the equation $y'' + k^2y = g(x)$, where $k \neq 0$ and $g(x)$ is continuous on I is given by $y(x) = A\cos kx + B\sin kx + \frac{1}{k}\int_0^x \sin k(x-t)g(t)dt$. (3)

4. (a) Solve $(D^2 - 4)y = x\sin hx$. (4)
   (b) Find the values of a, b and c such that the directional derivative of $\phi = axy^2 + byz + cz^2x^3$ at $(1,2,-1)$ has a maximum magnitude 64 in the direction parallel to $z-$ axis. (3)
   (c) Find the work done by the force $F = z\bar{i} + x\bar{j} + y\bar{k}$ along the curve C consists of line segments from $(0,0,0)$ to $(1,2,3)$ and then from $(1,2,3)$ to $(3,2,5)$. (3)

5. (a) Use spherical coordinates to find the volume of the solid that lies above the cone $z = \sqrt{x^2 + y^2}$ and below the sphere $x^2 + y^2 + z^2 = z$. (5)
   (b) Find the volume bounded by the cylinder $x^2 + y^2 = 4$ and the hyperboloid $x^2 + y^2 - z^2 = 1$. (5)

6. (a) Find the surface area which is part of the hemi sphere $x^2 + y^2 + z^2 = 2, z \geq 0$ above the cylinder $x^2 + y^2 = 1$. (5)
   (b) Verify Green's theorem in plane for $\oint_C (x^2 - 2xy)dx + (x^2y + 3)dy$ where C is the boundary of the region bounded by $y^2 = 8x$ and $x = 2$. (5)

7. (a) Evaluate $\oint_C 2y^3dx + x^3dy + zdz$ where C is the trace of the cone $z = \sqrt{x^2 + y^2}$ intersected by the plane $z = 4$ and S is the surface of the cone below $z = 4$ (5)
   (b) Verify the divergence theorem for $A = 4x\bar{i} - 2y^2\bar{j} + z^2\bar{k}$ taken over the region bounded by $x^2 + y^2 = 4, z = 0$ and $z = 3$. (5)