Visvesvaraya National Institute of Technology, Nagpur
Department of Mathematics
Re-Examination 2014

Subject: Mathematics-I (MAL101)

Max. Marks: 60 **Time: 3:00 hrs**
Note: Section A is compulsory. Answer any five questions from Section B.

Section A $[5 \times 2 = 10]$
Answer any five Questions from Section A.

1. (a) Let f be a real valued function of real variable and $a \in \mathbb{R}$. If f differentiable at 'a, evaluate the

$$\lim_{x \to a} \frac{xf(a) - af(x)}{x - a}.$$

(b) Find a point on the curve $f(x) = \sqrt{x-2}$ in $[2, 3]$ where the tangent is parallel to the chord joining the end points.

(c) Let A be a square matrix with eigen value λ. Find λ if $A^4 = I$.

(d) Test the convergence of the series $\sum_{n=1}^{\infty} \frac{(n+1)^n}{n^{n+5/4}}$.

(e) Test the convergence of the improper integral $\int_a^b \frac{dx}{(x-a)^p} dx \quad (p > 0)$.

(f) Using $(\varepsilon - \delta)$ definition show that $\lim_{n \to \infty} \frac{1}{n} = 0$.

Section B

2. (a) State and prove Taylor's theorem with remainder. [4]
(b) Find the center and circle of curvature of the curve $xy(x + y) = 2$ at $(1, 1)$. [3]
(c) Briefly discuss tracing of the curve $r = a\cos 2\theta,\ a > 0$. [3]

3. (a) Determine the area between the curves $y = x^3$ and $y = 4x^3$. [3]
(b) Find the volume of the solid generated by revolving the finite region bounded by the curves $y = x^2 + 1$, $y = 5$ about the line $x = 3$. [4]
(c) Show that $\beta(m, n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$. [3]

4. (a) Consider the quadratic form $3x^2 + 4xy + 3y^2 = Q(\mathbf{x})$. Find the orthogonal matrix S such that $Q(\mathbf{x})$ transforms to its canonical form and hence find the canonical form. [3]
(b) Is the matrix $A = \begin{bmatrix} -1 & -4 \\ 3 & -2 \end{bmatrix}$ diagonalizable? If so find the non-singular matrix P such that $P^{-1}AP$ is diagonal matrix. [4]
(c) Verify the Cayley-Hamilton theorem for the matrix $A = \begin{bmatrix} 1 & 2 & 0 \\ -1 & 1 & 2 \\ 1 & 2 & 1 \end{bmatrix}$. [3]

5. (a) Test for conditional convergence of the series $\sum_{n=1}^{\infty} (-1)^n (\sqrt{n-1} - \sqrt{n})$. [4]
(b) Discuss the convergence of the series $\sum_{n=1}^{\infty} \frac{n}{n^2 + 10}$. [3]

(c) Test for convergence of the series $\sum_{n=2}^{\infty} \frac{\ln n}{n^p} \quad (p > 0)$. [3]

6. (a) Find the radius of convergence and interval/circle of convergence of the following power series

   (i) $\sum_{k=1}^{\infty} \frac{(x-3)^k}{k2^k}$. [4]

   (ii) $\sum_{n=1}^{\infty} n^{\ln n} z^n$. [3]

   (b) Test for convergence of the improper integral $\int_{-1}^{1} \frac{x}{(x^2-1)^2} dx$. [3]

7. (a) Discuss the convergence of $\int_{0}^{1} x^{m-1}(1-x)^{n-1} dx$. [4]

   (b) Test for convergence of the improper integral $\int_{0}^{1} \frac{\sin(1/x)}{x^p} dx$. [3]

   (c) Test for convergence of the improper integral $\int_{-\pi/2}^{\pi/2} \tan x dx$. [3]

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