Visvesvaraya National Institute of Technology, Nagpur
Department of Mathematics

Subject: Mathematics-I (MAL101) Re-Examination 2013-2014

Max. Marks: 60 **Time: 3:00 hrs**

NOTE:**Answer any five Questions from Section A.**
Answer any five Questions from Section B.
Maximum marks are indicated against each Question.
Calculators are not permitted.

Section A $[5 \times 2 = 10]$

1. (a) Test the consistency of the system of equations

$$5x + 3y + 7z = 4; \quad 3x + 26y + 2z = 9; \quad 7x + 2y + 10z = 5.$$

(b) Evaluate $\int_0^{\pi/2} \sqrt{\tan x}\, dx$, using Beta function.

(c) Is $(4, 5, 5)$ a linear combination of $(1, 2, 3)$, $(-1, 1, 4)$ and $(3, 3, 2)$? Justify your answer.

(d) Verify Rolle's theorem for the function $\frac{\sin x}{e^x}$ in $[0, \pi]$.

(e) Show that the function $f(x) = \begin{cases} \frac{xe^x}{1+e^x} & \text{if } x \neq 0; \\ 0 & \text{if } x = 0 \end{cases}$ is not differentiable at $x = 0$.

(f) Test the convergence of the series $\sum_{n=1}^{\infty} \frac{\ln n}{n^2}$

Section B

2. (a) Verify whether the matrix $A = \begin{bmatrix} 5 & -6 & -6 \\ -1 & 4 & 2 \\ 3 & -6 & -4 \end{bmatrix}$ is diagonalizable or not? If so, find the modal matrix. [5]

(b) Show that the all eigen values of a Hermitian matrix are real. [3]

(c) Let A be a $n \times n$ matrix. Show that A^T and A have same eigen values. [2]

3. (a) Find the area of the region bonded by the curves $y = \sin x$, $y = \cos x$ from $x = 0$ to $x = \pi/2$. [5]

(b) Find the volume of the solid obtained by rotating the region bounded by $y = 2x^2 - x^3$ and $y = 0$ about the y-axis. [5]

4. (a) Evaluate $\int_0^{\infty} \frac{e^{-ax} \sin x}{x}\, dx$ by differentiation under the sign of integration. Hence find $\int_0^{\infty} \frac{\sin x}{x}\, dx$.[5]

(b) Show that $\int_0^{\infty} \frac{x^c}{c^x}\, dx = \frac{\lceil c+1}{(\ln c)^{c+1}}, \quad c > 1.$ [3]

(c) Show that $\beta(m, n) = \beta(m + 1, n) + \beta(m, n + 1)$. [2]

5. (a) Determine the values of a, b, c so that the function $f(x) = \begin{cases} \dfrac{\sin(a+1)x + \sin x}{x} & \text{if } x < 0; \\ c & \text{if } x = 0; \\ \dfrac{\sqrt{x + bx^2} - \sqrt{x}}{bx^{3/2}} & \text{if } x > 0 \end{cases}$

is continuous for all x. [3]

(b) Show that $1 + x < e^x < 1 + xe^x$ for all $x > 0$. [3]

(c) State and prove Taylor's theorem with Lagrange's form of remainder. [4]

6. (a) Show that the radius of curvature of the ellipse $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$ at the end of major axis is equal to the semi latus rectum of the ellipse. [3]

(b) Find the radius and circle of convergence of the power series $\sum\limits_{n=1}^{\infty} n^{\log n} z^n$. [3]

(c) Test the convergence of the integral $\int\limits_0^1 x^{m-1}(1-x)^{n-1}dx, \;\; 0 < m < 1, \; 0 < n < 1.$ [4]

7. (a) Show that the sequence $\left\{\dfrac{2n+3}{3n-7}\right\}$ converges. Find its limit. [3]

(b) Show that the series $\sum\limits_{n=1}^{\infty} \left(\dfrac{\sin n}{n}\right)^n$ converges absolutely. [3]

(c) Trace the curve $y^2x^2 = x^2 - a^2$. [4]

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