Max Marks: 10   Date:$19 - 08 - 2013$

Duration: 1 hour(9.00 AM – 10.00 AM)

Note: Answer any FIVE questions. All questions carry equal marks.

Calculators are not permitted.

1. If $f(x) = x^2 \sin \frac{1}{x}$ when $x \neq 0$ and $f(0) = 0$, show that $f(x)$ is derivable for every value of x but the derivative is not continuous for $x = 0$.

2. Let $f : [a, b] \to R$ be a differentiable function. Using Lagrange's mean value theorem show that $f(x)$ is constant if and only if $f'(x) = 0$ for every $x \in [a, b]$.

3. State Rolle's theorem and give its geometric interpretation. Also check the applicability of Rolle's theorem for $f(x) = 1 - x^{\frac{2}{3}}$ in $[-1, 1]$.

4. Define rank of a matrix and find the rank of $\begin{bmatrix} 1 & -2 & 0 & 3 & 2 \\ 5 & -5 & 1 & -5 & 2 \\ 2 & 1 & 1 & 1 & -4 \\ 4 & -3 & 1 & 7 & 0 \end{bmatrix}$.

5. (a) Is the system $x + y - z = 1, x + y + z = -1$ consistent? If so, find its solution(s)?
   (b) Suppose x_1 and x_2 are two solutions of $Ax = 0$, then show that $k_1 x_1 + k_2 x_2$ is also a solution of $Ax = 0$ for any $k_1, k_2 \in R$.

6. Determine whether the vectors $V_1 = (1, 4, 7)^T$, $V_2 = (2, 5, 8)^T$, $V_3 = (1, 2, 3)^T$ are linearly dependent or linearly independent. If they are linearly dependent then find a relation between them.