

Visvesvaraya National Institute of Technology, Nagpur
Department of Mathematics
I Semester B.Tech. Second Sessional Examinations
Mathematics (MAL 101)

Max Marks: 15

Date: 17 – 10 – 2014

Duration: 1 hour (9.00 AM – 10.00 AM)

Note: Answer any FIVE questions. All questions carry equal marks.

Calculators are not permitted.

1. State and prove first fundamental theorem of integral calculus.
2. The line segment $x = \sin^2 t, y = \cos^2 t, 0 \leq t \leq \frac{\pi}{2}$ is revolved about the Y - axis. Find the surface area of the solid generated.
3. Evaluate $\int_0^{\frac{\pi}{2}} \frac{dx}{\sqrt{1 - \frac{1}{2} \sin^2 x}}$.
4. Use Leibnitz's rule for differentiation under integral sign to evaluate $F(2)$ from $F(\alpha) = \int_0^{\infty} \frac{e^{-x} - e^{-\alpha x}}{x \sec x} dx, \alpha > 0$.
5. Define consistency and inconsistency for the system of linear equations. For what values of k the equations $x + y + z = 1, 2x + y + 4z = k, 4x + y + 10z = k^2$ have a solution and solve them completely in each case.
6. Prove that the vectors $\begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix}, \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} -3 \\ 7 \\ 2 \end{bmatrix}$ are linearly dependent. Also find the relation among them.