

Visvesvaraya National Institute of Technology, Nagpur
Department of Mathematics
I Semester B.Tech. Second Sessional Examinations
Mathematics (MAL 101)

Max Marks: 20

Date: 30 – 09 – 2013

Duration: 1 hour (9.00 AM – 10.00 AM)

Note: Answer any FIVE questions. All questions carry equal marks.

Calculators are not permitted.

1. State Taylor's theorem with Cauchy form of remainder. Verify Taylor's theorem for $f(x) = x^3 - 3x^2 + 2x$ in $[0, \frac{1}{2}]$ with Lagrange's remainder up to two terms.
2. (a) Evaluate $\lim_{x \rightarrow 0} \frac{(1+x)^{1/x} - e + \frac{ex}{2}}{x^2}$.
(b) Trace the curve $y^3 = x^2(6 - x)$.
3. Trace the curve $r^2 = a^2 \sin 2\theta$. Also find the radius of curvature at any point (r, θ) of the curve $r^2 = a^2 \sin 2\theta$.
4. Verify Cayley - Hamilton's theorem and hence find A^{-1} if $A = \begin{pmatrix} 1 & i & i \\ i & 1 & i \\ i & i & 1 \end{pmatrix}$.
5. Check for the diagonalizability of the matrix $A = \begin{bmatrix} 1 & 1 & -1 \\ -1 & 1 & 1 \\ -1 & 1 & 1 \end{bmatrix}$ and find the modal matrix if exists.
6. (a) Define quadratic form. Also find the matrix of the quadratic form $2x^2 - y^2 - z^2 + 4xy + 8yz - 4xz$.
(b) Find the area under the curve $y = 4 - x^2$ bounded by the x - axis.