Visvesvaraya National Institute of Technology, Nagpur
Department of Mathematics
Subject: Mathematics-I (MAL-101)
End Semister Examination

Duration: 3 hours Max. Marks:60
Date: 01-12-2014

Note: i) Answer any **five** questions from **Section A** and answer any **five** questions from **Section B**.
ii) Calculators are not permitted .

Section-A **5×2=10**

1. (a) Find length of the curve $x = \cos^3 t, y = \sin^3 t, 0 \leq t \leq 2\pi$.

   (b) Find the area of the region that lies inside the circle $r = a\cos\theta$ and outside the cardioid $r = a(1 - \cos\theta)$.

   (c) Show that the eigen values of a Hermitian matrix are always real.

   (d) If the system $A^2X = 0$ has non-trivial solution then show that $AX = 0$ also has a non-trivial solution.

   (e) Show that the sequence $\{a_n\}$ converges, where

   $$a_n = \frac{1}{\sqrt{1+n^2}} + \frac{1}{\sqrt{2+n^2}} + ... + \frac{1}{\sqrt{n+n^2}}.$$

   (f) Discuss the convergence of the improper integral $\int_0^1 \frac{1}{\sqrt{1-x^3}}dx$.

Section-B **5X10=50**

2. (a) Does the function $f(x) = \begin{cases} -x^2, -1 \leq x \leq 0 \\ x^2, 0 \leq x \leq 1 \end{cases}$ differentiable? If so find its derivative. (Give justification for your answer). (3)

   (b) State and prove Cauchy's mean value theorem. (4)

   (c) Find the equation of the circle of curvature for the curve $xy = 1$ at (1,1). (3)

3. (a) Trace the curve $x = a(t + \sin t), y = a(1 - \cos t)$. (4)

   (b) The area between the curves $y = 2x - x^2$ and $y = x$ is revolved about $y-$ axis to generate a solid. Find the volume of the solid generated. (3)

   (c) Show that (3)

   $$\int_0^\infty \frac{x^c}{c^x}dx = \frac{\Gamma(c+1)}{(\ln c)^{c+1}}, \text{ where } c > 1.$$

4. (a) Check whether the matrix $\begin{pmatrix} 3 & 2 & 1 \\ 0 & 2 & 0 \\ 1 & 2 & 3 \end{pmatrix}$ is diagonalizable or not. (4)

(b) Using Cayley-Hamilton theorem, find the inverse of the matrix $\begin{pmatrix} -1 & -1 & 1 \\ 1 & -1 & 1 \\ 0 & 1 & 1 \end{pmatrix}$. (3)

(c) Find the quadratic form of the matrix $\begin{pmatrix} 1 & 2 \\ 0 & 3 \end{pmatrix}$ and hence find the associated canonical form of this quadratic form. (3)

5. (a) Discuss the convergence of the infinite series (3)

$$\sum_{n=1}^{\infty} \frac{4^n (n!)^2}{(2n)!}.$$

(b) Find the radius of convergence and the interval of convergence of the power series (4)

$$\sum \frac{(-3)^n}{\sqrt{n+1}} (x-1)^n.$$

(c) Discuss the convergence of the series (3)

$$\sum_{n=1}^{\infty} \left(\frac{3n-2}{np+1} \right)^n (3-4i)^n.$$

6. (a) Is the series $\sum 2^{2n} 3^{1-n}$ convergent? Justify your answer. (2)

(b) Test for convergence of the infinite series (3)

$$\sum_{n=2}^{\infty} \left(\frac{1}{n(\ln n)^p} \right), (p > 0).$$

(c) Show that the integral $\int_0^{\infty} e^{-x^2} dx$ converges. (2)

(d) Is the improper integral $\int_3^{\infty} \frac{dx}{x^2 + x - 2}$ converges if so find its limit. (3)

7. (a) Test for convergence of (3)

$$\int_{-\infty}^{\infty} \frac{\cos x}{1+x^2} dx.$$

(b) Discuss the convergence of the improper integral (3)

$$\int_1^2 \frac{\sqrt{x}}{\ln x} dx.$$

(c) Determine the value of p for which the the following improper integral converges (4)

$$\int_0^{\infty} \frac{1 - e^{-x}}{x^p} dx.$$