Visvesvaraya National Institute of Technology, Nagpur
Department of Mathematics
I Semester B.Tech. End Semester Examination
Mathematics I (MAL 101)

Max Marks: 60 Date:18 –11 – 2013

Duration: 3 hour(9.00 a.m.– 12.00 noon)

Note: (i) Section A is compulsory.
(ii) Answer any Five questions from Section A .
(iii) Answer any Five questions from Section B.
(iv) Maximum marks are mentioned in the brackets for each question.
(v) Calculators are not permitted.

Section A $5 \times 2 = 10$

1. (a) Evaluate $\underset{x \to \infty}{Lim} \left(\frac{\sinh x}{x}\right)^{\frac{1}{x^2}}$.
(b) If a 3×3 matrix A has eigen values $1, 2, -1$, then find the trace of the matrix $B = A - A^{-1} + A^2$.
(c) Evaluate $\int_0^\infty 2^{-9x^2} dx$ using Gamma function.
(d) Calculate the length of the arc of the cycloid $x = a(t - \sin t), y = a(1 - \cos t)$ between two cusps.
(e) Test for convergence of $\sum_{n=2}^{\infty} \frac{1}{nlog_e n}$.
(f) Evaluate the improper integral $\int_{-1}^{1} \frac{dx}{x^2}$ if it exists.

Section B

2. (a) Show that the function $f(x) = \begin{matrix}(x-1)\tan\frac{\pi x}{2}, x \neq 1 \\ -1, x = 1\end{matrix}$ is not differentiable at $x = 1$. (3)
(b) State and prove Lagrange's mean value theorem. Also give its geometrical interpretation. (4)
(c) Find the circle of curvature of the curve $x + y = ax^2 + by^2 + cx^3$at the origin. (3)

3. (a) Determine the common area between the circle $r = \frac{3}{2}a$ and the cardioid $r = a(1 + \cos\theta)$. Also find the area which is external to the circle but inside the cardioid. (5)
(b) Find the volume of the solid generated by revolving the region bounded by the curves $y = 1 + \sqrt{x}$ and $y = 1 + x$ about the Y-axis. (5)

4. (a) Prove that $\int_a^b (x-a)^{m-1}(b-x)^{n-1}dx = (b-a)^{m+n-1}\beta(m,n)$, where m, n, a, b are positive constants. (5)
(b) Using Leibnitz rule of differentiation under integral sign, evaluate the integral $\int_0^\infty e^{-x^2}\cos(2\alpha x)dx$ where α is the parameter. (5)

5. (a) For what values of k the equations $x + y + z = 1, 2x + y + 4z = k, 4x + y + 10z = k^2$ have a solution and solve them completely in each case. (5)
(b) Find the orthogonal transformation which transforms the quadratic form $x^2+3y^2+3z^2-2yz$ to canonical form. (5)

6. (a) Trace the curve $r = 2(1 - 2\sin\theta)$. (4)
   (b) Determine the approximate value of π using Maclaurin's series expansion of $\sin^{-1} x$. (3)
   (c)Test for the convergence of the series $\sum_{n=1}^{\infty} \frac{1.4.7.....(3n-2)}{2.5.8.....(3n-1)}$. (3)

7. (a) Test for the conditional convergence of $\frac{1}{2^3} - \frac{1}{3^3}(1+2) + \frac{1}{4^3}(1+2+3) - \frac{1}{5^3}(1+2+3+4) + ...\infty$. (4)
   (b) Show that the power series $\sum_{n=1}^{\infty} \frac{z^{2n}}{4^n n^\alpha}, \alpha > 0$ is convergent in the region $|z| < 2$. (3)
   (c) Determine the value of p for which $\int_0^\infty \frac{1-e^{-x}}{x^p} dx$ converges. (3)