Assignment 1

MAL 407 (Sampling Theory and Estimation Theory)

1. Construct Sampling distribution of means for the population 3, 7, 11, 15 by drawing samples of size two with replacement. Determine (a) μ (b) σ (c) Sampling distribution of mean (SDM) (d) $\mu_{\bar{x}}$ (e) $\sigma_{\bar{x}}$, verify results.

2. Solve the above problem if sampling is without replacement.

3. Determine the mean and standard deviation of Sampling distribution of variances for the population 3, 7, 11, 15 with n =2 and with sampling (a) with replacement (b) without replacement.

4. Find standard error of sample means of size 2 drawn from a population 2, 3, 6, 8, 11 with replacement and without replacement.

5. Calculate the probability that a random sample of 16 computers will have an average life of less than 775 hours assuming that length of life of computers is approximately normally distributed with mean 800 hours and s.d. 40 hours.

6. Determine the probability that $\bar{x}$ will be between 75 and 78 if a random sample of size 100 is taken from an infinite population having the mean $\mu = 76$, $\sigma^2 = 256$.

7. Assume that the heights of 3000 male students at an university are normally distributed with mean 68.0 inches and s.d 3.0 inches. If 80 samples consisting of 25 students each are obtained, what would be the expected mean and s.d. of the resulting sampling distribution of means if the sampling were done (a) with replacement and without replacement?

8. Find the value of the finite population correction factor for n= 10, and N = 1000.

9. Find the probability that of the next 200 children born
    a. Less than 40% will be boys
    b. Between 43% and 57% will be girls.
    c. More than 54% will be boys.

    Assume equal probabilities for births of boys and girls.

10. Find the probability that in 120 tosses of a fair coin between 40% and 60% will be heads.

11. Let $U_1 = \{2, 7, 9\}$, $U_2 = \{3,8\}$. Find
    (a) μ_{u_1} (b) μ_{u_2} (c) $\mu_{u_1+u_2}$ (d) $\mu_{u_1-u_2}$ (e)σ_{u_1} (f) σ_{u_2} (g) $\sigma_{u_1+u_2}$ (h) $\sigma_{u_1-u_2}$ (i) $\mu_{u_1-u_2} = \mu_{u_1} - \mu_{u_2}$ (j) $\mu_{u_1+u_2} = \mu_{u_1} + \mu_{u_2}$ (k) $\sigma_{u_1 \pm u_2} = \sqrt{\sigma_{u_1} + \sigma_{u_2}}$

12. The mean life time of light bulbs produced by a company is 1500 hours and s.d. of 150 hours. Find the probability that lighting will take place for (a) at least 5000 hours (b) at most 4200 hours if three bulbs are connected such that when one bulb burns out, another bulb will go on. Assume that life times are normally distributed.

13. Company A and B manufacture two types of cables, having mean breaking strengths of 4000 N and 4500 N and standard deviations of 300 N and 200 N, respectively. Determine the probability that the mean breaking strength of cables produced by company B will be (a) at least 600 N more than (b) atleast 450 N more than the cables produced by company A, if 100 cables of brand A and 50 cables of brand B are tested.

14. Let $\bar{x}_A$ and $\bar{X}_B$ be the average drying times of two types of paints A and B, for samples of size $n_A = n_B = 18$. Suppose $\sigma_A = \sigma_B = 1$. Find $P(\bar{x}_A - \bar{X}_B > 1.0)$. Assuming that the mean drying time is equal for that two types of paints.

15. Find the t value with $\nu = 14$ dof that leaves an area 0.025 to left.

16. Find P (- t0.025 < t < t0.05).

17. Find k such that P (k <t < -1.761) =0.045 for a random sample of size 15 selected from a normal distribution.

18. A sample of 12 measurements of the breaking strengths of cotton threads gave a mean of 7.38 oz and a standard deviation of 1.24 oz. Find 95 % and 99% confidence limits oz actual mean breaking strength. Assuming that the methods of large sampling theory are applicable, compare the results obtained.

19. In a random sample of 400 adults and 600 teenagers who watched a certain television program, 100 adults and 300 teenagers indicated that they liked it. Construct 95% confidence limits for the difference in proportions of all adults and all teenagers who watched the program and liked it.

20. The Standard deviation of the lifetimes of 10 electric light bulbs manufactured by a company is 120 hours. Find 95% confidence limits for the standard deviation of all bulbs manufactured by the company.