Visvesvaraya National Institute of Technology, Nagpur
Department of Mathematics

Assignment-4 **Subject: MAL-205**

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1. Using power method find the largest eigen value and the corresponding eigen vector of the matrices
(i) $\begin{bmatrix} 4 & 1 & 0 \\ 1 & 20 & 1 \\ 0 & 1 & 4 \end{bmatrix}$ (correct to three decimal places) (ii) $\begin{bmatrix} 2 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 2 \end{bmatrix}$.

2. Calculate an approximation to the least eigen-value of $A = LL^t$, where $L = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$ using one step of inverse power method. Choose the vector $(6, -7, 3)^t$ as a first approximation to the corresponding eigenvector.

3. Find the eigen-value correct to two decimal palces which is nearest to 5 for the matrix $\begin{bmatrix} 4 & 1 & 0 \\ 1 & 4 & 1 \\ 0 & 1 & 4 \end{bmatrix}$ using inverse power method. Obtain the corresponding eigen-vector. Take the inial approximation vector as $(1, 1, 1)^t$.

4. Solve the equation $y' = \dfrac{1}{x^2} - \dfrac{y}{x} - y^2, y(1) = -1$ from $x = 1$ to $x = 2$. Use Taylor 's series of order 2 and order 4, with $h = \dfrac{1}{4}$.

5. Using Taylor's series, find the solution of the differential equation $xy' = x - y$, $y(2) = 2$ at $x = 2.1$ correct to five decimal places.

6. From the Taylor's series for $y(x)$, find $y(0.1)$ correct to six decimal places if $y(x)$ satisfies
$$y' = xy + 1,\ y(0) = 1.$$

7. Solve the following problems using Euler's method with step size $h = 0.2$.
(i) $y'(x) = (\cos y(x))^2, 0 \le x \le 2,\ y(0) = 0$; (ii) $y' = \dfrac{1}{4}y\left(1 - \dfrac{1}{20}y\right), 0 \le x \le 2,\ y(0) = 1.$

8. Consider the problem
(i) $y' = -10y + 11\cos x + 9\sin x, y(0) = 1,$
(ii)$y' = -10y + \dfrac{1}{1+x^2} + 10\tan^{-1} x, y(0) = 0,$
Solve the problem using modified Euler's method with $h = \dfrac{1}{5}$ for $y(1)$.

9. Solve problem 4, 7 and 8 using Runge-Kutta second and fourth order methods.