Visvesvaraya National Institute of Technology, Nagpur
Department of Mathematics

Assignment-3 **Subject: MAL-205**

1. Let $Ax = b$ (for arbitrary b) (i) if $A = \begin{bmatrix} 1 & -k \\ -k & 1 \end{bmatrix}$, $k \in \mathbb{R}$, then determine k such that Gauss Jacobi and Gauss Seidel method converges, (ii) if $A = \begin{bmatrix} 1 & k \\ 2k & 1 \end{bmatrix}$, $k \in \mathbb{R}$ and $k \neq \frac{\sqrt{2}}{2}$, find the value of k for which the Gauss Jacobi method converges.

2. Find the necessary and sufficient condition on k, so that the (i) Gauss-Jacobi iterative method, (ii) Gauss-Seidel iterative method converges for solving the system of equations $Ax = b$, where $A = \begin{bmatrix} 1 & 0 & k \\ 2 & 1 & 3 \\ k & 0 & 1 \end{bmatrix}$ and b is arbitrary.

3. Discuss the convergence of the methods (i) Gauss-Jacobi iterative method, (ii) Gauss-Seidel iterative method for solving the system of equations $Ax = b$, and hence solve the system, where $A = \begin{bmatrix} 1 & 2 & -2 \\ 1 & 1 & 1 \\ 2 & 2 & 1 \end{bmatrix}$ and $b = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$.

4. Write Gauss-Jacobi iteration method in matrix form for the system of equations $\begin{aligned} 3x_1 + x_2 + x_3 &= 2 \\ x_1 + 4x_2 + 2x_3 &= -5 \\ x_1 + 2x_2 + 5x_3 &= 2 \end{aligned}$ and find the largest eigen value of the iterative matrix (use Newton Raphson method to solve the characteristic equation). Hence find the rate of convergence of the iterative method.

5. Solve the system of equations $\begin{array}{rrrrl} 2x & -y & & & = 1 \\ -x & +2y & -z & & = 0 \\ & -y & +2z & -w & = 0 \\ & & -z & +2w & = 1 \end{array}$ using Gauss Jacobi and Gauss Seidel iterative methods taking initial guesses $x^{(0)} = (\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2})^T$ and $x^{(0)} = (0, 0, 0, 0)^T$ respectively (perform three iterations in each case).

6. Solve $\begin{array}{rrrrl} 4x & +y & & & = 2 \\ x & +4y & +z & & = -2 \\ & y & +4z & +w & = 2 \\ & & z & +4w & = -2 \end{array}$ using Gauss Jacobi iterative method with zero vector as an initial guess.

7. Using Jacobi method find all eigen values and the corresponding eigen vectors of the matrices (i) $\begin{bmatrix} 3 & 2 & 2 \\ 2 & 5 & 2 \\ 2 & 2 & 3 \end{bmatrix}$ (ii) $\begin{bmatrix} 2 & \sqrt{2} & 4 \\ \sqrt{2} & 6 & \sqrt{2} \\ 4 & \sqrt{2} & 2 \end{bmatrix}$ (iii) $\begin{bmatrix} 3 & 2 & 1 \\ 2 & 3 & 2 \\ 1 & 2 & 3 \end{bmatrix}$. (In (i) for first rotation use a_{13} as a largest off diagonal element, for (iii) Iterate till the off-diagonal elements in magnitude are less than 0.005).