Visvesvaraya National Institute of Technology, Nagpur
Department of Mathematics

Assignment-1 **Subject: MAL-205**

1. Can we solve the equation $\tan x = 0$ in the interval $[1.5, 1.6]$.
2. The Dipstick equation
$$3\cos^{-1}(1-h) - 3(1-h)\sqrt{2h-h^2} - 1 = 0$$
has a root near to $h = 0.3$, find the root using Bisection method.
3. Use Bisection method to obtain the root of the equation
$$x^3 + 4x^2 - 10 = 0$$
which lies in the interval $[1, 2]$ correct to one decimal place.
4. How many steps would you except to have to carry out to solve the equations.
   (i) $e^x - 3x = 0, a = 0, b = 1$
   (ii) $x^3 - 3x + 1 = 0, a = 1, b = 2$

   correct to two decimal places,using the starting intervals given?
5. Verify that the equation
$$0.5\sin x - x + 3 = 0$$
has a root in the interval $[0, 2\pi]$. Choose a suitable iterative formula to locate this root correct to two decimal places.
6. Show that, if any of the following iterative formulae converge, they will converge to the root of the equation $x^2 - 6x + 5 = 0$
   (i) $x_{n+1} = 6 - \dfrac{5}{x_n}$
   (ii) $x_{n+1} = \dfrac{x_n^2 + 5}{6}$
   (iii) $x_{n+1} = \sqrt{6x_n - 5}$

   without carrying out any iterations, determine whether or not any of the formulae do converge to either root,assuming that a suitable starting value is chosen.
7. Determine whether or not the iterative formula
$$x_{n+1} = x_n^2 - \frac{1}{x_n}$$
with a suitable starting value,would converge to the root of the equation $x^3 - x^2 - 1 = 0$, which is near to 1.5.
8. Obtain the value of $3\sqrt{25}$ correct to two decimal places using both the Bisection method and the Newton Raphson method. Note that the number of iterations required in each case.
9. Consider the equation $f(x) = 0$ where
$$f(x) = (x-1)^3(x-2) = x^4 - 5x^3 + 9x^2 - 7x + 2$$
Evaluate the two roots correct to three decimal places,using the Newton Raphson method, using $x_0 = 0.5, 1.5$ and 2.5. Note that the number of iterations required for convergence in each case and comment on the result. Use also Newton Raphson method for multiple roots.

10. Find the smallest root of the equation

$$1 - x + \frac{x^2}{(2!)^2} - \frac{x^3}{(3!)^2} + \frac{x^4}{(4!)^2} - \frac{x^5}{(5!)^2} + \cdots = 0$$

correct to two decimal places, using the iteration method.

11. The equation $\sin x = 5x - 2$ can be written as $x = \sin^{-1}(5x - 2)$ and also as $x = \frac{1}{5}(\sin x + 2)$,suggesting two iterating procedures for its solution. Which of these, if either, would succeed, and which would fail to give a root in the neighborhood of 0.5.

12. If α, β are the roots of $x^2 + ax + b = 0$. Show that the iteration $x_{n+1} = -\left(\frac{ax_n + b}{x_n}\right)$ will converge near $x = \alpha$ if $|\alpha| > |\beta|$ and the iteration $x_{n+1} = \frac{-b}{x_n + a}$ will converge near $x = \alpha$ if $|\alpha| < |\beta|$.

13. State and prove the sufficient condition for convergence of iteration method.

14. Find a real root of the equation $x \log_{10} x = 1.2$ by Regula-Falsi method correct to four decimal places.

15. Compute the root of the equation $x^2 e^{-x/2} = 1$ in the interval $[0, 2]$ using the Secant method.The root should be correct to three decimal places.

16. Find the real root of the equations by the Newton-Raphson method $F(x, y) = 0.2x^2 + 0.8 = 0$ and $G(x, y) = 0.3xy^2 + 0.7 = 0$

17. Find the cube root of 15 correct to four significant figures by iteration method.

18. Derive the rate of convergence and asymptotic error constant of the Secant method.

19. Determine the value of p and q so that the rate of convergence of the iterative method

$$x_{n+1} = px_n + q\frac{N}{x_n^2}$$

for computing $N^{1/3}$ becomes as high as possible.