Visvesvaraya National Institute of Technology,Nagpur
Department of Mathematics
Assignment-2

1. Find the radius of curvature, centre of curvature and circle of curvature for the given curve at the indicated point
   (a) $x^3 + y^3 = 3axy$ at $\left(\frac{3a}{2}, \frac{3a}{2}\right)$
   (b) $y = c \ln \sec(\frac{x}{c})$ at (x, y)
   (c) $x^{(2/3)} + y^{(2/3)} = a^{(2/3)}$ at (x, y)
   (d) $x^2 y = a(x^2 + y^2)$ at $(-2a, 2a)$
   (e) $xy^2 = a^3 - x^3$ at $(a, 0)$
   (f) $\sqrt{x} + \sqrt{y} = \sqrt{a}$ at $(\frac{a}{4}, \frac{a}{4})$
2. Find the point where radius of curvature is minimum for $x^2 y = a\left(x^2 + \frac{a^2}{\sqrt{5}}\right)$.
3. If R_1 and R_2 are radii of curvature at the extremities if a focal chord at a parabola $y^2 = 4ax$. Then prove that $R_1^{(-2/3)} + R_2^{(-2/3)} = (2a)^{(-2/3)}$
4. For the curve $y = \dfrac{ax}{(a+x)}$ prove that $\left(\dfrac{y}{x}\right)^2 + \left(\dfrac{x}{y}\right)^2 = \left(\dfrac{2R}{a}\right)^{2/3}$
5. Prove that $R_1^{(2/3)} + R_2^{(2/3)} = \dfrac{(a^2 + b^2)}{(ab)^{(2/3)}}$, where R_1 and R_2 are radii of curvature at the extremities of the conjugate diameters of an ellipse $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$.
6. Find the radius of curvature for the curves whose equations are given in polar form:
   (a) $r = a(1 + \cos\theta)$
   (b) $r^n = a^n \sin n\theta$
   (c) $r^2 \cos 2\theta = a^2$
   (d) $r = 2\cos 2\theta$ at $\theta = \pi/6$
7. Find the radius of curvature, centre of curvature and circle of curvature for the curves whose equations are given in parametric form:
   (a) $x = \ln t \; y = \dfrac{1}{2}\left(t + \dfrac{1}{t}\right)$
   (b) $x = a(t + \sin t)$, $y = a(1 - \cos t)$
   (c) $x = a\ln(\sec t + \tan t)$, $y = a \sec t$
   (d) $x = 2\cosh t$, $y = 2\sinh t$ at $t = 0$
   (e) $x = a\cos^3 t$, $y = a \sin^3 t$
   (f) $x = a\cos t$, $y = b \sin t$
8. If R_1 and R_2 be the radii of curvature at the extremities of any chord of $r = a(1 + cos\theta)$ which passes through the pole, then prove that $R_1^2 + R_2^2 = \dfrac{16}{9}a^2$

9. Prove that for the curve $r^2 = a^2 \sin 2\theta$ curvature varies as the radius vector.

10. Find the point on the parabola $y^2 = 8x$ at which the radius of curvature is $125/16$.

11. Find the radius of curvature at $(0, 0)$

    (a) $y^4 + x^3 + a(x^2 + y^2) - a^2 y = 0$
    (b) $x^3 + y^3 - 2x^2 + 6y = 0$
    (c) $2x^4 + 3y^4 + 4x^2 y + xy - y^2 + 2x = 0$
    (d) $x^3 + 3x^2 y - 4y^3 + y^2 - 6x = 0$

12. Trace the following curves whose equations are given in Cartesian form.

    (a) $y^2(a - x) = x^3$
    (b) $a^2 x^2 = y^3(2a - y)$
    (c) $4ay^2 = x(x - 2a)^2$
    (d) $y^2(a + x) = x^2(3a - x)$
    (e) $ay^2 = x(a^2 - x^2)$
    (f) $y^2(a^2 - x^2) = a^3 x$
    (g) $y(x^2 + a^2) = a^3$
    (h) $3ay^2 = x^2(a - x)$
    (i) $ay^2 = x(x^2 + a^2)$
    (j) $x^2 y^2 = a^2(y^2 - x^2)$

13. Trace the following curves whose equations are given in Polar form.

    (a) $r^2 = a^2 \sin 2\theta$
    (b) $r = a(1 + \cos\theta)$
    (c) $r = 2(1 - 2\sin\theta)$
    (d) $r = a + b\cos\theta$
    (e) $r = a\sin 2\theta$
    (f) $r = a\cos 3\theta$
    (g) $r = a\sin 3\theta$
    (h) $r = a\cos 2\theta$
    (i) $r^2 = a^2 \cos 2\theta$
    (j) $r = 1 + \sqrt{2}\cos\theta$

14. Trace the following curves whose equations are given in Parametric form.

    (a) $x = a(\theta - \sin\theta)$, $y = a(1 - \cos\theta)$
    (b) $x = a(\theta + \sin\theta)$, $y = a(1 - \cos\theta)$
    (c) $x = e^t + e^{-t}$, $y = e^t - e^{-t}$
    (d) $x = a\cos^3 t$, $y = b\sin^3 t$