**Assignment 1**

Course: Numerical Solution of Differential Equation (MAL-402)
Branch: M.Tech. (W.R.E)

1. Solve the following linear boundary value problem using the shooting method. Write the corresponding initial value problem as a system of two first order initial value problem and solve it by the given method.

$$u''=2x^{-2}\,u-x^{-1}, \qquad 2<x<3$$

$$u(2)=0, \quad u(3)=0.$$

2. Solve the following nonlinear boundary value problem using the shooting method.

$$u''=2uu', \quad 0<x<1$$
$$u(0)=1/3, \qquad u(1)=0.5.$$

3. Use the Taylor series method of second order to solve the initial value problems and the secant method for iteration. Assume $h$=0.25 and two approximations to the slope as $s^{(0)}$=0.05, $s^{(1)}$=0.2. Perform two iterations. Compare the results with the exact solution $u(x)=\frac{1}{3-x}$.
4. Determine the values of $y$ at [0, 1]. If y satisfies the BVP

$$y^{iv}(x)+81y=81\,x^2,$$

$$y(0)=y(1)=y''(0)=y''(1)=0, \qquad \text{take } n=3.$$

5. The deflection of the beam is governed by the equation

$$\frac{d^4y}{dx^4}+81y=\emptyset(x)$$

Where $\emptyset(x)$ is governed by the table

| $x$ | = | 1/3 | 2/3 | 1 |
|---|---|---|---|---|
| $\emptyset(x)$ | = | 81 | 162 | 243 |

and the boundary condition $y(0)=y'(0)=y''(1)=y'''(1)=0$.

6. Given the boundary value problem $x^2y''+xy'-y=0$, $y(1)=1$ and $y(2)=0.5$ apply the cubic spline method to determine the value of $y(1.5)$.
7. Solve $y''=2yy'$ with $y(0)=0.5$, $y'(0)=0.25751$.
8. Find the approximate solution of the BVP

$$y''+(8\,sin^2\pi x)y=0, \quad 0\le x\le 1$$

$$y(0)=y(1)=1 \text{ take (n=4)}$$

9. Solve the following BVP by using $2^{nd}$ order central difference formula

$y''+xy=1$

$y(0)=0$, $y'(1)=1$.

10. Solve the following BVP $y''+xy=1$

$y(0)+y'(0)=1$, $y(1)=1$

11. Consider the BVP defined by

$y''+\frac{4x}{1+x^2}y'+\frac{2}{1+x^2}y=0$,

with $y(0)=1$, $y(2)=0.2$.

12. Use the shooting method to solve the BVP

$u''=2uu'$ $0< x <1$

$u(0)=0.5$ and $u(1)=1$.

13. Solve the BVP $u''=4u+3x$ $0<x<1$

$u(0)=1$, $u(1)=1$

with $h=0.25$ using (i) the second order method (ii) Numerov method Compare the results with the exact solution values.

14. Solve $u''-3u'+2u=0$, $0<x<1$,

$u'(0)-u(0)=1$

$u'(1)+u(1)=1$ with step size $h=0.25$

15. Solve $u''=6u^2$ $0<x<0.3$

$u(0)=1$, $u(0.3)=100/169$,

use the Taylor series method of order four to solve the initial value problems and the secant method for iteration. Assume $h=0.1$ and two approximations to the slope as $s^{(0)}=-1.8$, $s^{(1)}=-1.9$. Iterate till $|\emptyset(s)|<0.005$. Compare with the exact solution u(x) = $\frac{1}{(1+x)^2}$ .