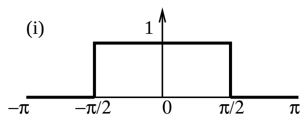
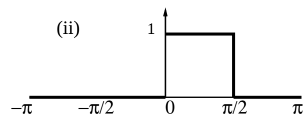
VISVESVARAYA NATIONAL INSTITUTE OF TECHNOLOGY, NAGPUR
DEPARTMENT OF MATHEMATICS

Topic: Assignment-1(Fourier Series) **MAL-201, B. Tech., III-Semester.**

1. Find the Fourier series for the functions with period 2π, shown in the following figures:





2. Find the Fourier series of the following periodic functions, whose period is 2π:
(i) $f(x) = e^{-ax}$ for $-\pi \le x < \pi$, (ii) $f(x) = x + |x|$ for $-\pi < x < \pi$,
(iii) $f(x) = \begin{cases} x, & -\frac{\pi}{2} < x < \frac{\pi}{2} \\ \pi - x, & \frac{\pi}{2} < x < \frac{3\pi}{2} \end{cases}$ (iv) $f(x) = |\cos x|$ for $-\pi \le x \le \pi$.

3. Find the Fourier series of the function f defined in the interval $[-\pi/2, \pi/2]$ as $f(x) = \cos x$.

4. Find the Fourier series of the following $2L$ periodic functions:
(i) $f(x) = 1 - x^2, -1 < x < 1$, (ii) $f(x) = \begin{cases} \frac{1}{2} + x, & -\frac{1}{2} < x < 0 \\ \frac{1}{2} - x, & 0 < x < \frac{1}{2} \end{cases}$, (iii) $f(x) = \begin{cases} x, & 0 < x < 1 \\ 1 - x, & 1 < x < 2 \end{cases}$.

5. An alternating current after passing through a rectifier has the form $I(t) = \begin{cases} I_0 \sin t, & 0 \le t \le \pi \\ 0, & \pi \le t < 2\pi \end{cases}$, where I_0 is the maximum current. Express $I(t)$ as a Fourier series.

6. Find the Fourier series of the periodic function that is obtained by passing the voltage $V(t) = v_0 \cos 100\pi t$ through a half-wave rectifier.

7. Find the Fourier series for $f(x) = \begin{cases} 2, & -\pi < x < 0 \\ -2, & 0 < x < \pi \end{cases}$. Explain the Gibb's phenomenon.

8. Show that an even function can't have sine terms in its Fourier series.

9. Find the half-range Fourier cosine and sine expansions of the following functions:
(i) $f(t) = t, 0 < t < 1$ (ii) $f(t) = t^2, 0 < t < 3$ (iii) $f(t) = \begin{cases} \frac{2k}{L} t, & 0 < t < \frac{L}{2} \\ \frac{2k}{L}(L - t), & \frac{L}{2} < t < L \end{cases}$
(iv) $f(t) = e^t, 0 < t < L$ (v) $f(t) = t - t^2, 0 < t < 1$.

10. Find the Fourier series of $f(x) = \frac{x^2}{2}, -\pi < x < \pi$ and $f(x + 2\pi) = f(x), \forall x$. Hence show that
(a) $\frac{\pi^2}{6} = 1 + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + ...$ (b) $\frac{\pi^2}{12} = 1 - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + ...$

11. If $f(x) = \begin{cases} 0 & \text{for } -\pi \le x < 0 \\ \sin x & \text{for } 0 \le x < \pi \end{cases}$, prove that $f(x) = \frac{1}{\pi} + \frac{\sin x}{2} - \frac{2}{\pi} \sum \frac{\cos 2n\pi}{4n^2 - 1}$. Hence show that,
$\frac{\pi - 2}{4} = \frac{1}{1.3} - \frac{1}{3.5} + \frac{1}{5.7} - \frac{1}{7.9} + ...$

12. If $f(x) = \cos \mu x$, on $[-\pi, \pi]$, where $\mu$is not an integer. Deduce that $\cot \mu\pi = \frac{2\mu}{\pi} + \frac{1}{\mu^2 - 1^2} + \frac{1}{\mu^2 - 2^2} +$ .... Hence show that $\sum \dfrac{1}{9n^2 - 1} = \dfrac{1}{2} - \dfrac{\pi\sqrt{3}}{18}$.

13. Find the Fourier coefficients corresponding to the function $f(x) = \begin{cases} 0 & \text{for } -5 < x < 0 \\ 3 & \text{for } 0 < x < 5 \end{cases}$, $f(x+10) = f(x), \forall x.$
How should $f(x)$ be defined at $x = 0, \pm 5$ in order that the Fourier series will converges to $f(x)$ for $-5 \leq x \leq 5$?

14. Find the complex Fourier series of the following functions :
(i) $f(x) = \begin{cases} 1, & -\pi < x < 0 \\ -1, & 0 < x < \pi \end{cases}$ $f(x+2\pi) = f(x)\ \forall x.$) (ii) $f(t) = e^{-|x|}, -2 \leq x < 2$

(iii) $f(x) = \begin{cases} x, & 0 \leq x < 1 \\ 2-x, & 1 \leq x < 2 \end{cases}$, $f(x+2) = f(x)\ \forall x.$ (iv) 

15. Find the complex Fourier series of the full wave rectification of $E_0 \sin(\lambda t)$ where E_0 and λ are positive constants (period $2L = 2\pi/\lambda$).

16. Find frequency spectrum of the following periodic pulses:
(i) $f(x) = \begin{cases} 1, & -1 < x < 0 \\ -1, & 0 < x < 1 \end{cases}$, $f(x+2) = f(x), \forall x,$
(ii) $f(x) = \begin{cases} 0, & -\frac{\pi}{2} \leq x < 0 \\ \sin x, & 0 \leq x < \frac{\pi}{2} \end{cases}$ $f(x+\pi) = f(x), \forall x.$

17. Show that $\frac{L}{2} - x = \frac{L}{\pi}\sum_{j=1}^{\infty} \frac{1}{j} \sin\left(\frac{2j\pi x}{L}\right),\ 0 < x < L.$

18. Let $f(x) = \begin{cases} x + \frac{\pi}{2}, & \text{if } -\pi \leq x < 0 \\ -x + \frac{\pi}{2} & \text{if } 0 \leq x \leq \pi \end{cases}$. Is this function even or odd?
Find the Fourier series of this function and use it to evaluate the sum of the series $1 + \frac{1}{3^2} + \frac{1}{5^2} + \ldots\ldots.$

19. Let a function $f(x)$ is defined only over the range $0 \leq x \leq 2$ to be $f(x) = x$. Find half-range Fourier sine seires of $f(x)$? Does it converges to the function at the end point $x = 2$? If not, extend this function in such a way that its Fourier sine series will converges to the value of the function at $x = 2$. Justify your answer.

20. Consider the function f defined by $f(x) = \begin{cases} 0 & \text{for } -\pi \leq x < 0 \\ x+1 & \text{for } 0 \leq x < \pi/2 \\ 2x & \text{for } \pi/2 \leq x < \pi \end{cases}$ on the interval $-\pi \leq x < \pi$ and for all other x by the periodicity condition $f(x+2\pi) = f(x)$ for all x. Discuss the convergence of the Fourier series of f. In particular, determine the value to which the series converges at each of the points $x = 0, x = \pi/2$ and $x = \pi$.