Visvesvaraya National Institute of Technology, Nagpur
Department of Mathematics
Assignment-2

Multiple Integrals **Subject: MAL-102**

1. If $\int_0^8 \int_{x^{\frac{1}{3}}}^2 dydx = \int_a^b \int_c^d dxdy$, then find a, b, c and d.
2. Prove that for all $x > 0$,
$$3\int_0^x \int_0^u u^2 f(t)dtdu = \int_0^x (x^3 - u^3)f(u)du.$$
3. Evaluate
$$\iint_A \left[2a^2 - 2a(x+y) - (x^2+y^2)\right] dydx,$$
where A is the region inside the circle $x^2 + y^2 + 2ax + 2ay - 2a^2 = 0$.
4. Find the y coordinate of the center of gravity of the region bounded by $y = \sqrt{4-x^2}, y = \sqrt{9-x^2}$ and the $x-$axis.
5. Let C be the cone generated by revolving line $y = \dfrac{x}{\sqrt{3}}$ starting from $(0,0)$ around $y-$axis. Let S be the solid sphere $x^2 + y^2 + z^2 = 4$. Find the volume of the portion of S that lies inside C.
6. The base of a certain solid is the region between $x-$axis and the curve $y = \sin x$ between $x = 0$ and $x = \dfrac{\pi}{2}$. Each section of the solid perpendicular to the $x-$axis is an equilateral triangle with one side in the base of the solid. Find the volume of the solid.
7. Evaluate
$$\int_0^1 \int_0^\pi \int_x^\pi \frac{\sin y}{y} dydxdz.$$
8. If R is the region between the spheres $x^2 + y^2 + z^2 = a^2$ and $x^2 + y^2 + z^2 = b^2, b > a$. Evaluate
$$\iiint_R (x^2 + y^2)dzdydx.$$
9. Find the volume bounded by the cylinder $x^2 + y^2 = 4$ and the hyperboloid $x^2 + y^2 - z^2 = 1$.
10. Evaluate
$$\iiint_V x^2y^2z^2 dzdydx$$
where V is the volume bounded by $xy = 4, yz = 1, xz = 25, xy = 9, yz = 4$ and $xz = 49$.
11. Prove that by changing the order of integration
$$\int_0^x \int_0^v \int_0^u e^{m(x-t)} f(t)dtdudv = \int_0^x \frac{(x-t)^2}{2} e^{m(x-t)} f(t)dt.$$