Assignment 1

Course: Numerical Methods (MAL-202) **Branch: Civil/Mining Engg.**

1. State and prove existence and uniqueness theorem of interpolating polynomials.

2. What is inverse interpolation?

3. Establish Newton's divided difference formula and give an estimate of the remainder term in terms of the appropriate derivative. Deduce Newton's forward and backward interpolation formula as particular cases.

4. Obtain error formulas for Lagrange's interpolation formula.

5. Derive the formula for Hermite interpolation.

6. The function f(x) =$Sinx$ is defined on the interval [1, 3],

(i) Obtain the Lagrange's linear interpolating polynomials in this interval and find the bound on the truncation error. Obtain the approximate values of f (1.5) and f (2.5).

(ii) Divide the interval [1, 3] into two subinterval [1, 2] and [2, 3]. Obtain the Lagrange's linear interpolating polynomial in each subinterval and find the bound on the truncation error. Hence find the approximation values of f (1.5) and f (2.5).

7. Find the error in Hermite interpolating polynomial.

8. Using Newton's forward difference formula find the sum of

S_n= 1^3+ 2^3+3^3+…….+n^3.

9. Derive the formula for the first derivative of y=f(x) of O (h^2) using

(i) Forward difference approximation

(ii) Backward difference approximations

(iii) Central difference approximation

When $f(x) = Sinx$, estimate f'($\frac{\pi}{4}$) with h=$\frac{\pi}{12}$ using the above formulas. Obtain the bounds on the truncation error and compare with the exact solution.

10. Given the following values of f(x) =$Logx$, find the approximate value of f'(2.0) using linear and quadratic interpolation and f'' (2.0) using quadratic interpolation. Also obtain an upper bound on the error

i	0	1	2
x_i	2.0	2.2	2.6
y_i	0.69315	0.78846	0.95551

11. For linear interpolation equispaced data tabular data, show that the error does not exceed $\frac{1}{8}$ of the second difference.

12. Find the third-order Hermite polynomial passing through the points (x_i, y_i, y'_i), i=0, 1.

13. Tabulate y=x^3 for x=2, 3, 4 and 5 and calculate the cube root to three decimal places.

14. Find the quadrature formula

$\int_0^1 \frac{f(x)}{\sqrt{x(1-x)}} dx$=$\alpha_1$f(0)+ α_2f(1/2)+ α_{31}f(1)

Which is exact for polynomial of highest possible degree? Then use the formula on

$$\int_0^1 \frac{dx}{\sqrt{x - x^3}}$$

And compare with the exact value.

15. Derive Gauss –Legendre integration formula.

16. Determine the step size h that can be used in the tabulation of a function$f(x)$, a$\leq x \leq b$, at equally spaced nodal points so that the truncation error of the quadratic interpolation is less than ε.