VISVESVARAYA NATIONAL INSTITUTE OF TECHNOLOGY, NAGPUR
DEPARTMENT OF MATHEMATICS
Assignment 1I

Course: Numerical Solution of Differential Equation (MAL-402**)** **Branch**: M.Tech. (W.R.E)

1. Derive the truncation error of the following methods
   (i) Laasonen method
   (ii) Crank Nicholson method
   (iii) Richardson's method
   (iv) Dufort-Frankel method
2. Use the Schmidtz scheme to find the numerical solution of following initial BVP
   (i) $u_t = u_{xx}$
   $u(x, 0) = 100(x - x^2)$, $0 < x < 1$
   $u(0, t) = u(1, t) = 0$ take $h = 1/4$, $k = 1/32$.

   (ii) $u_t = 4\, u_{xx}$
   $u(0,t) = 0 = u(8,t)$
   and $u(x, 0) = 4x - \frac{1}{2}x^2$; $h = 1$, $k = 1/8$.
3. Solve $u_t = u_{xx}$
   $u(x, 0) = Sin\pi(x)$, $0 \leq x \leq 1$
   $u(0,t) = u(1,t) = 0$
   (i) Using Crank-Nicholson method
   (ii) Using Dufort- Frankel method

4. Use Crank-Nicholson method and Central differences to solve the initial value problem
   $u_t = u_{xx}$, $u(x, 0) = 1$, $0 \leq x \leq 1$
   $(\frac{\partial u}{\partial x})_{(0,t)} = u(0, t)$;
   $(\frac{\partial u}{\partial t})_{(1,t)} = -\, u(1, t)$; $t > 0$, with step length $h = 1/3$ and $\lambda = 1/3$.

5. Derive the stability condition for Crank Nicholson and Richardson schemes.

6. Solve the initial value problem
   $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}$
   $u(x, y, 0) = Cos(\frac{\pi x}{2}) - Cos(\frac{\pi y}{2})$, $-1 \leq x, y \leq 1$, $t = 0$
   $u = 0$, $x = \pm 1$, $y = = \pm 1$, $t > 0$. Using the Peaceman Rachford ADI method with $h = 1/2$ and $r = 1/6$.

7. Compute u for 4 time step using explicit 2nd finite difference method given that
   $u_{tt} = u_{xx}$;
   $u(0, t) = u(x, 0) = \frac{\partial u}{\partial t}(x, 0) = 0$
   $u(1, t) = 100\, sin\pi t$ with $h = 1/k = 1/4$.

8. Find The solution of the initial boundary value problem
   u_{tt}=u_{xx} ; $0 \leq x \leq 1$, such that the initial condition
   $u(x, 0)= sin\pi x$, $0 \leq x \leq 1$
   $\frac{\partial u}{\partial t}(x, 0)$ =0, $0 \leq x \leq 1$
   $u(0, t) = u(1, t)$=0 with $t > 0$. By using explicit scheme. Assume h=1/4, r=3/4. Integrate for 5 time step.

9. Find the eigen values and eigen vectors of a common tridiagonal matrix.
10. Discuss the convergence condition for Crank – Nicholson scheme.